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Our Responsibility to Change the World
The Mathematics and Philosophy of Chaos

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Our Responsibility to Change the World: The Mathematics and Philosophy of Chaos

I. Abstract

The study of chaos theory has altered our understanding of how the world works, forcing us to consider philosophical questions with new knowledge. From a mathematical perspective, chaos theory reveals phenomena that contradict mathematical intuition while teaching us how to deal with uncertainty and complexity in the world. The questions surrounding free will and determinism have long been debated by philosophers and have real consequences in everyday life. The mathematics of chaos theory and philosophical thought on free will and determinism are significant in their respective fields, but taken together can teach us about the responsibility we have for our thoughts and actions and the effects of our choices on the world around us. The purpose of this project is to survey key concepts of chaos theory from a mathematical perspective and consider some of the stances on the free will debate. By integrating knowledge of the mathematics and philosophy related to chaos theory, we can feel empowered to take responsibility for our actions and know that our choices do, in fact, make a difference.

II. Introduction to Chaos Theory

Chaos theory has changed the way we think about problems. Chaos differs from other sciences and their trend toward reductionism—reducing systems to smaller parts, like cells or atoms, and studying how these parts function—by focusing on the whole (Gleick 5). It deals with the problems and solutions that scientists may have written off as incoherent or unsolvable. Within the challenging problems and incomprehensible solutions—that is, within chaos—we can find patterns and "subtle relationships between simplicity and complexity and between orderliness and randomness" (Hall 7). The study of chaos not only reveals interesting order and patterns in surprising places, it also provides a way of working with uncertainty and complexity. For this reason, chaos theory often naturally gives rise to more philosophical questions about our place in the world.

Chaotic systems all share several characteristics. Chaotic systems are sensitively dependent on their initial conditions (Smith 1). Similar systems that begin with similar initial conditions will diverge until they no longer resemble each other. For this reason, sensitive dependence is often associated with instability (16). Chaotic systems are also deterministic and nonlinear (1). These characteristics mean that the behavior of the system is completely determined by its current state. In the short term, we can used the current state to predict future states of the system. But long term predictions become impossible because of nonlinearity; small changes in the system or errors in our predictions will compound to force our predictions to diverge from the true state of the system (Percival 13). Chaos "does not imply that prediction is hopeless," merely that as the predictions forecast states further ahead in time, the uncertainty increases as well (Smith 87). But because chaotic systems are deterministic, short term predictions can be made with more accuracy. The three characteristics of chaotic systems—sensitive dependence, determinism, and nonlinearity—produce results that defy mathematical intuition, which is precisely what makes chaos such an interesting field.

The study of chaos changes our perspective on the process of scientific research. Chaos theory attempts to deal with the results that other sciences may have written off because of their complexity or incomprehensibility. Chaos shows that "complicated-looking solutions are sometimes acceptable and need not be due to external dynamic noise" (Smith 74). Furthermore, "apparently patternless behavior may become simple and comprehensible if you look at the right picture" (Stewart 58). The solutions that have previously seemed unusable actually reveal interesting patterns about dynamic systems. Researchers just had to find the best perspective or representation of the data to find the patterns. This shows that above all, chaos theory can teach us about the relationships between uncertainty, complexity, disorder, and order that can be found in so many systems in the world around us.

Chaos can be approached from a variety of disciplines and its results can be applied to a number of fields. Current research related to chaos theory affects medicine, population biology, astronomy, engineering, and economics (Hall 9; Smith 71). Chaos helps researchers in these fields find meaning in noisy data—such as the regularity and irregularity of heart beats or noise in message transmission—and understand change in dynamic systems—such as populations of animals or prices of cotton (Gleick 86, 91-92; Hall 8-10). Chaos invites an interdisciplinary approach. For this reason, the results from researching chaos from a mathematical perspective are more meaningful when contextualized with perspectives from another field.

In order to get a conceptual understanding of some big themes in chaos theory, we will discuss several concepts—such as nonlinearity and attractors—and experiment with some mathematical dynamical systems that demonstrate chaotic behavior. Once we have surveyed these topics, we will contextualize these results to understand what chaos theory can teach us about responsibility for our actions and the effect we have on our surroundings.

III. Introduction to Free Will and Determinism

Free will is the idea that our thoughts and actions are up to us, meaning they are under our direct control. Determinism is the concept that everything is caused by previous events. In the context of human behavior, determinism means that our decisions and actions are determined by our past experiences. Free will and determinism are concepts that philosophers have considered for a long time. Do we have free will? That is, are our thoughts and actions up to us? Or are our thoughts and actions determined, meaning they are the effects of specific causes? Are free will and determinism necessarily contradictory concepts?

Intuitively, it seems that free will and determinism are incompatible with each other. Our decisions are not up to us if these decisions are determined by our genes, experiences, or some greater cosmic event. This intuition about our free will leads to a school of thought known as incompatibilism. Within incompatibilist thought, there are further distinctions. Some philosophers—such as Robert Kane—divide incompatibilism into libertarianism and determinism, while others—such as Thomas Pink in his book, *Free Will: A Very Short Introduction*—distinguish between libertarianism and skepticism. Libertarians believe that determinism is false and we are free (Kane, *Contemporary* 32; Pink 19). Determinists believe that free will is impossible because our behavior is deterministic (Harris 15; Kane, *Contemporary* 32). Skeptics generally do not believe in the existence of free will. But skepticism serves as a broad category for any incompatibilist perspective that does not quite align with libertarianism or determinism. One example of a skeptical perspective, which will be discussed in a later section of the paper, is the belief that freedom is inconsistent with both determinism and indeterminism, making free will impossible (Pink 19). For the purposes of this project, the focus will be placed on libertarianism and determinism. However, skepticism will be briefly discussed in order to obtain a more complete understanding of the variety of stances on the free will debate.

The other stance on the free will debate is compatibilism. Compatibilists believe that free will and determinism are consistent ideas (Pink 19). According to compatibilism, we are free and the world is deterministic. This perspective reconciles our intuition about our freedom along with progress in both the physical and social sciences that generally supports determinism (Kane, *Contemporary* 12). Though quantum theory in physics refutes the idea of universal determinism, other sciences—such as biology, biochemistry, neuroscience, psychiatry, psychology, and other social and behavioral sciences—seem to support a form of causal determinism, even for human behavior (10). The compatibilist perspective proposes that our intuition about our freedom and scientific evidence that seems to explain our behavior through deterministic causes—such as our genes and past experiences—can coexist. Our behaviors can be determined and we can have free will at the same time.

The significance of free will and determinism relates to our understanding of several concepts that are central to the functioning of society, such as ethical questions of responsibility. For example, our legal justice system relies on the assumption that individuals are responsible for their thoughts, intentions, and actions. If we are not free to make our own decisions, how do our code of ethics and our legal justice system need to change to reflect our lack of freedom? Our understanding of ethical questions of responsibility, which is often taken for granted in our everyday lives, requires reflection on the philosophical questions surrounding free will and determinism.

For the purposes of developing a conceptual understanding of the major stances surrounding the free will debate, some of the nuances of the arguments used to defend incompatibilism—further divided into libertarianism, determinism, and skepticism—and compatibilism will be explored. These perspectives will be briefly evaluated in the context of this project and then integrated with examples from chaos theory in order to consider the significance of these perspectives and their contributions to ethical questions of responsibility. Philosophical thought on the free will debate—from the works of popular author Sam Harris and philosopher Robert Kane, among other thinkers—will be discussed in later sections of the paper.

IV. Nonlinearity

Now that we have established a brief conceptual understanding of the mathematics and philosophy related to the study of chaos theory, we will develop more detailed discussions of several concepts and demonstrations related to the mathematics of chaos.

Chaos is a science that acknowledges and works with nonlinearity. Mathematically, nonlinearity occurs when components of the variables that represent the current state of the system are multiplied together (Smith 38). This means that the current state of the system affects future states in ways that are difficult to predict. “Nonlinearity means that the act of playing the game has a way of changing the rules. […] That twisted changeability makes nonlinearity hard to calculate, but it also creates rich kinds of behavior that never occur in linear systems” (Gleick 24). Nonlinear systems are challenging yet interesting to work with because they create feedback that affects the future behavior of the system.

The behavior of such systems seems counterintuitive. Linear systems demonstrate predictable behavior; we can perform operations and expect the system to change proportionally to the operation. Nonlinear systems do not respond proportionally (Smith 10). For example, in a nonlinear system, large changes could damp out to nothing while small changes blow up to have dramatic effects on the long term behavior of the system. Students learn how to work with systems that behave linearly and can be modelled with solvable equations. This develops a mathematical intuition that the world is largely linear. However, the behavior of most systems is nonlinear and modelled by unsolvable equations (Gleick 68). It may seem surprising how common nonlinearity and chaotic behavior are in the world around us. For this reason, chaos seems to contradict mathematical intuition. Though the results can be difficult to understand, the mathematics of chaos reveals complexity in such systems, and may even demonstrate a certain amount of order.

The nonlinearity of chaotic systems contributes to the sensitive dependence of these systems. Nonlinear equations introduce exponential growth of uncertainty into the system. Uncertainty in the measurements of the initial conditions of the system will grow exponentially or compound. This means that the prediction of each future state becomes more uncertain as we try to forecast further into the future (Smith 24). Because of the sensitive dependence and nonlinearity of chaotic systems, "similar states diverge under deterministic dynamics" (51). Even under similar initial conditions, the behavior of the same system would quickly diverge in each case until the behaviors seem to be completely independent and distinct. Chaotic behavior is a result of the interaction between these characteristics.

Though nonlinear systems behave in complicated ways that differ from what our mathematical intuition would expect, they also demonstrate rich and interesting behaviors. Nonlinearity will play an important role in the chaotic behavior of mathematical dynamical systems explored in later sections of the paper.

V. Attractors

Another common theme in chaos theory is the idea of an attractor. Chaos theory studies dynamic systems, which are systems that change. A natural question when studying change is if the system ever settles into some stable or periodic behavior or if the system perpetually moves between unstable states. This question relates to the attractor, or "the inevitable if unreachable destination" (Smith 37) of the behavior of the system.

Attractors represent the eventual behavior of dynamic systems under all possible initial conditions. Many chaotic systems have attractors that describe the evolution of their behavior in the long term, whether they are mathematical dynamical systems that settle into states represented by numbers or physical dynamical systems that settle into certain patterns of motion. In either case, the range of possible behaviors—numbers or paths—is referred to as the phase space. In his article "Portraits of Chaos," Ian Stewart explains the relationship between attractors and the behavior of chaotic systems:

> No matter where you start from, if you wait long enough, the system will follow this single trajectory to as high a degree of approximation as you wish. Any region of phase space with this property is known as an attractor. Usually, a system does not explore the whole of phase space: instead, the dynamical laws pick out small regions, and almost all of the long-term motion takes place solely within those regions. Any point that starts outside those regions is "attracted" to them. (Stewart 50)

The intriguing property about attractors is that any initial condition for the chaotic system will eventually result in behavior within or infinitesimally close to the system's attractor.

There are several different types of attractors. Two types of attractors that are relatively easy to visualize—specifically in the context of mathematical dynamical systems—are fixed point attractors and periodic motion attractors. Fixed point attractors correspond to a single point in the phase space. The behavior of the system will evolve to be arbitrarily close to the fixed point attractor, eventually settling into steady-state behavior (Smith 37). Periodic motion attractors represent closed loops—a finite series of points or paths—that regularly repeat themselves (Stewart 50-51). We will further discuss fixed point and periodic attractors in the next section.

Slightly more complicated attractors are referred to as multidimensional tori, which combine distinct periodic motions whose periods are not necessarily related. A multidimensional torus represents quasiperiodic motion, which "almost repeats, but never quite gets back to the exact starting gate" (Stewart 51). An example of a multidimensional torus would be one that represents orbits in our galaxy: the moon moves periodically around the Earth, which moves periodically around the Sun, which moves periodically through the galaxy (52). This series of periodic orbits combine to form quasiperiodic motion. A visualization of quasiperiodic motion from Stewart's article can be seen in Appendix A.

Still more complicated attractors are referred to as strange attractors. Strange attractors represent more complex motion than other types of attractors. A significant amount of research goes into characterizing and analyzing the dynamics of strange attractors. Given the frequency with which examples of periodic motion appear when studying physics and differential equations, it may seem hard to believe that any sort of motion would ever need to be described using a strange attractor. Stewart comments on the how the existence of strange attractors is possible:

> Did classical mathematics miss something? Indeed it did. Classical mathematics had rather limited tools. It wanted solutions that could be specified by a tidy formula, so it therefore concentrated on equations that could easily be solved by a tidy formula. Unfortunately, most cannot, and within this silent majority here lurk innumerable attractors of a distinctly less cozy form than your friendly neighborhood torus. They are called strange attractors. That does not mean they are in any way unusual; indeed the only thing unusual about them is that they are unusually common. It means that nobody understands them very well. (52)

The types of motion represented by strange attractors are actually quite common, despite our mathematical intuition that we live in a linear or periodic world.

Strange attractors not only represent complicated motions, but they also represent infinite complexity. Strange attractors are fractal, meaning they have "complete structure on any scale of magnification" (Stewart 52). This means that if we zoom in on an image of a strange attractor, no matter how far we zoom in, we will be able to find infinitely many possible paths. Under some initial conditions, the behavior of the system will eventually follow any possible trajectory within the shape of the strange attractor.

One of the most well-known examples of a strange attractor is the Lorenz attractor, which was first depicted by meteorologist Edward Lorenz. In 1960, Lorenz created a model for the weather using three equations and twelve parameters (Gleick 11-12). Even though his model simplified physical weather systems, he noticed some interesting and surprisingly realistic behaviors in his system. In 1961, Lorenz wanted to rerun part of a simulation and input the values from a computer printout into the equations. He found that the resulting weather patterns quickly diverged from the patterns of the previous simulation until the two printouts did not resemble each other at all. Lorenz later realized that the values he input to his model were truncated by the computer for the printout, and differed from the actual parameter values for that point of the simulation by one thousandth (16). This dynamic nonlinear system and the differences between the two weather simulations represented sensitive dependence on initial conditions and chaotic behavior. Lorenz's weather model severely simplified real weather patterns, but nevertheless represented complex behavior representative of the physical system. This incident led to research—presented in his 1963 paper "Deterministic Nonperiodic Flow"—that greatly contributed to the development of the science of chaos.

By representing a state in his model with three numbers, Lorenz created a visualization of how his model changed over time within a three-dimensional phase space. This image, which has become a familiar image for chaos theory, is known as the Lorenz attractor, and represents the complex behavior of Lorenz's simplified weather model. Images of the Lorenz attractor can be seen in Appendix B. Like all strange attractors, "points that start close together get stretched apart as they circulate round the attractor," eventually following independent trajectories (Stewart 52). In addition, the Lorenz attractor has "infinitely many layers" (55) representing the infinite number of paths the system could eventually settle into under different initial conditions. Though there are many different types of strange attractors that have been found through research in chaos theory, the Lorenz attractor is one of the most recognizable.

Attractors are interesting features of chaos in that they represent patterns of behavior that can be found in what appears to be disorderly states of a system. In the next section, we will see a specific demonstration of attractors in a mathematical dynamical system.

VI. Feigenbaum Attractor

Chaos theory has helped researchers in numerous fields better understand dynamic systems in the context of their discipline and make sense of their findings. In particular, population biologists have learned more about how populations change from year to year based on chaos.

Changing population sizes represent dynamic systems. In order to model physical dynamical systems, researchers can use mathematical dynamical systems. Mathematical dynamical systems involve a rule or equation through which we iterate values of some variable (Smith 33). While we can represent the state of a physical dynamical system with numbers, the state of a mathematical dynamical system is the number itself (34). This distinction is subtle but significant in that mathematical dynamical systems necessarily simplify the real world problem into something that can be experimented with and studied. The behaviors in the mathematical dynamical system can be used to infer behaviors in the physical dynamical system, but not necessarily to represent or predict the behavior of the physical system exactly.

One equation used to model population growth is the logistic equation. The logistic equation is a nonlinear equation that finds a new population size (represented in the equation by variable x) based on the previous population size. The initial x value is chosen to be between zero and one and represents a percentage of the maximum population capacity supportable by the environment (Gleick 69-73). The logistic equation also uses a parameter λ—read lambda—which "quantifies the strength of the feedback" (Vivaldi 35). In other words, the parameter λ determines how much the previous value for x affects the new value of x. The logistic equation is:

$$x_{new} = \lambda * x_{old} * (1 - x_{old})$$

Once we have selected an initial x value, x_0, and a λ value, we can iteratively compute new values of x:

$$x_1 = \lambda * x_0 * (1 - x_0)$$

$$x_2 = \lambda * x_1 * (1 - x_1)$$

$$x_3 = \lambda * x_2 * (1 - x_2)$$

This iterative process can be continued as long as needed. We can experiment with different initial x values and different values of parameter λ to better understand the behavior of this mathematical dynamical system and make inferences about what this means in the context of population biology.

Suppose we select an initial x value and a value for λ. For a given x and λ, we can graph iterations of x through the logistic equation. To begin, we will consider $x_0 = 0.23$ and $\lambda = 1.50$. Computing 1,000 iterations, we get the graph in Appendix C. We can see that eventually, this system settles to a steady state around 0.33. This point represents a fixed point attractor for the system. In fact, the behavior of the system when λ = 1.50 for any initial value of *x* will be attracted to the point 0.33. To show that this is true, Appendix D shows the behavior of 1,000 iterations through the logistic equation with randomly chosen initial *x* values $x_0 = 0.45$ and $x_0 = 0.83$.

As we vary λ, interesting properties start to emerge from this system. With $x_0 = 0.23$ and λ = 2.99, the system has a fixed point attractor around 0.67. When we increase λ so that λ = 3.01, the system eventually settles into a periodic orbit between two points, approximately 0.63 and 0.70. The behavior of these two systems can be compared in Appendix E. If we let λ = 3.46, we can see four distinct settling values. The graph for this system is displayed in Appendix F. As λ increases, the period of the orbit of this mathematical dynamical system has doubled, from period one—a fixed point attractor—to period two, and from period two to period four. These regular increases in the period of the system are known as period doubling bifurcations (Smith 61). The increase in the parameter λ creates enough feedback to bring the system to a critical point, a bifurcation point, at which the long term behavior of the system changes.

As we have seen, the mathematical dynamical system under the logistic equation behaves in interesting ways. In order to get a more complete understanding of the long term behavior of this system, we can select a single initial *x* value and iterate *x* through the logistic equation while varying λ. In this computer simulation, we take $x_0 = 0.23$, let λ vary between 0 and 4 in increments of 0.0005, and compute 1,000 iterations of x_0 through the logistic equation to determine the eventual behavior of the system under the specific parameters. We can plot the settling values or attractors of the system against the corresponding values of λ. The resulting image is known as the Feigenbaum attractor—named after Mitchell Feigenbaum, a physicist who studied a variety of topics, including attractors and period doublings (Gleick 175)—as shown in Appendix G.

Looking at the Feigenbaum attractor reveals several noteworthy details about the eventual behavior of the logistic equation under different values of λ. First, the bifurcation points—points at which the period of the attractor doubles—get closer and closer together as λ increases. The periods begin doubling so quickly that eventually the system devolves into chaos, with no fixed point or periodic attractor (Smith 62, 64; Stewart 56). However, even in the midst of chaos, bands of order appear in the Feigenbaum attractor. The graph in Appendix H zooms in on the image of the Feigenbaum attractor from Appendix G. In Appendix H, we can see more clearly that the system transitions from chaotic behavior to a period three orbit. When looking at the graphs that plot the values of x against the number of iterations through the logistic equation, we can see that when $\lambda = 3.82$, the long term behavior of the system is chaotic and no periodic orbit can be identified. However, when $\lambda = 3.83$, the system settles into an orbit of period three. Appendix I compares these two mathematical dynamical systems.

The Feigenbaum attractor represents several recurring concepts in chaos theory. The logistic equation is a nonlinear equation, which makes its behavior difficult to predict. The parameter λ further complicates the behavior of the mathematical dynamical system by changing the amount of feedback. As λ increases, the previous x value has a greater effect on “correcting” the new x value (Vivaldi 35). Feedback complicates the iteration of the variable x through the system. Previous states effect future states in a way that is difficult to predict, and even small changes feed back into the system and contribute to the long term behavior. Enough feedback brings the system to a critical point that changes the eventual behavior of the system.

Furthermore, the Feigenbaum attractor is comprised entirely of attractors of the mathematical dynamical system as the parameter λ changes. For certain values of λ, the system converges to a fixed point attractor. As λ increases, a bifurcation point is reached and the attractor becomes a periodic orbit. Bifurcation points come closer together as λ increases, the period doubling each time, until the system devolves into chaotic behavior. But within the chaotic behavior, the system settles into structured periodic behavior. And out of this structure, period doubling bifurcations create chaos once again. The Feigenbaum attractor is a visual representation of the close relationship between disorder and order. Patterns can be found even through chaos.

The logistic equation and its long term behavior as represented in the Feigenbaum attractor allow population biologists to make inferences about physical dynamical systems. In fact, the conclusions drawn from the Feigenbaum attractor contradict much of what population biologists had thought about how populations fluctuate over time. Models such as the logistic equation are deterministic, but they can also be unpredictable due to feedback and nonlinearity (Vivaldi 40). The Feigenbaum attractor shows that these sorts of systems can fluctuate irregularly (Smith 59). Despite the erroneous assumption that "'natural' populations should maintain either a steady level or a regular periodic cycle" (Smith 59), population biologists have actually found populations—such as the Canadian lynx—that seem to fluctuate irregularly but according to "deterministic chaos" (Gleick 79). While it is important to distinguish between mathematical models and the actual system, mathematical dynamical systems often can inform useful inferences about the physical systems.

The Feigenbaum attractor embodies several significant features of chaos, including nonlinearity, attractors, feedback, and patterns that can be found within chaotic behavior. It also shows us the significance of mathematical dynamical systems in making inferences about the physical world.

VII. Fractals

Fractals are some of the most intriguing images of chaos. The word *fractal* was created in 1975 by Benoit Mandelbrot, who was described as a “mathematical jack-of-all-trades” for his work in fields such as economics, engineering, and mathematics (Gleick 83, 86, 98). Mandelbrot’s researched led to fractal geometry, which challenged geometers’ previous understandings of the world in terms of Euclidean shapes and dimension (94-98). As mentioned in the section on attractors, fractals have infinite complexity. These “mathematical objects” are self-similar on any scale (Smith 76, 79). It is important to differentiate between mathematical fractals and physical fractals. Mathematical fractals are truly infinitely complex. If we could zoom in on such an object with infinite precision, we could find similar copies of the original object on any scale (Smith 76). On the other hand, physical fractals have the property self-similarity on finite scales. Examples of physical fractals include trees and the circulatory system, both of which continue branching in a predictable way, but eventually stop branching at some scale (Gleick 108-110). Fractals are interesting representations of chaos because of their complexity in both mathematical and physical contexts.

One example of a mathematical fractal is the Sierpinski triangle, as seen in Appendix J. To form the Sierpinski triangle, start with an equilateral triangle. Connect the midpoints of all three sides, dividing the original triangle into four. Remove the middle triangle. In the three remaining triangles, connect the midpoints of all three sides and remove the middle triangle. This rule can be continued infinitely many times (Gleick 100). If we could zoom in to any part of the Sierpinski triangle, we would find more copies of a Sierpinski triangle. The Sierpinski triangle is a fractal because it is self-similar and displays infinite complexity on any scale.

Rather than creating the Sierpinski triangle conceptually, we can generate this fractal through a computer simulation. Mathematician Michael Barnsley developed a way to create the self-similar fractals from Mandelbrot's research through "the chaos game," which utilized randomness to create highly structured objects (Gleick 236-237). To play the chaos game that will generate the Sierpinski triangle, begin with three points arranged to be the endpoints of an equilateral triangle. Select any point within the perimeter of this triangle. From the current point, move half-way to a randomly selected endpoint, and plot a point. From this point, move half-way to a randomly selected endpoint, plotting the resulting point. This rule can be repeated as many times as needed (Barnsley 86-92). Notice that while the conceptual generation of the Sierpinski triangle can theoretically be repeated infinitely many times, creation through computer simulation can only be repeated a finite—if very large—number of times. Once again, we see the distinction between mathematical and physical fractals. However, in this case, the physical fractal does embody much of the complexity of the infinite mathematical fractal. The resulting image after 500,000 iterations of the rule from Barnsley's chaos game is displayed in Appendix K.

Though they can be generated by repeating simple rules, fractals push the boundaries of our imaginations and offer a peek at infinity. They invite us to consider mathematical and physical objects on infinitely large and infinitesimally small scales. They challenge us to change our perspective. Fractals represent the seemingly contradictory relationships between self-similar patterns and complexity that appear in many areas of chaos theory.

VIII. Incompatibilism

Now that we have discussed several key concepts and demonstrations of the mathematics of chaos, we will examine philosophical questions of free will and determinism. First, we will consider the different incompatibilist perspectives.

The incompatibilist perspectives all assert that free will and determinism are inconsistent concepts. If our decisions and actions are determined by some series of factors or events, then we cannot have free will. Our intuition generally supports incompatibilism:

> Our natural assumption is that our having control of how we act depends on our actions not being causally determined in advance by factors outside our control—by factors such as the environment we were born into, the genes we were born with, the desires and feelings that come over us beyond our control. (Pink 13)

We feel in control of our own thoughts and actions. If our thoughts or actions were causally determined, this would mean there were factors outside of our control affecting what we think and do. Our behavior would no longer be up to us. We tend to feel that this is not the case because we feel responsible for our thoughts, intentions, decisions, and actions. Therefore, if we truly have free will, then no external factors would cause us to behave in a particular way. Our intuition may not be enough to tell us whether or not we have free will. However, our general sense of our own freedom and the causes of our thoughts and actions justifies incompatibilism as a reasonable school of thought. But incompatibilists do not merely rely on our intuition to defend their stance on questions of free will.

Incompatibilists rigorously defend their perspective through a series of premises which construct the Consequence Argument. In his book *A Contemporary Introduction to Free Will*, Robert Kane discusses the Consequence Argument in depth, enumerating the premises and rules of logic that are used to jump from one premise to the next. Based on Kane's book, the premises and rules used by incompatibilists to construct the Consequence Argument are as follows:

1. "There is nothing we can now do to change the past."
2. "There is nothing we can now do to change the laws of nature."
3. "There is nothing we can now do to change the past and the laws of nature." Note that this premise merely combines the previous two premises.
4. "Our present actions are the necessary consequences of the past and the laws of nature. (Or, equivalently, it is necessary that, given the past and the laws of nature, our present actions occur.)"

   Rule Alpha: "There is nothing anyone can do to change what *must* be the case (or what is necessarily so)."
5. "There is nothing we can now do to change the fact that our present actions are the necessary consequences of the past and the laws of nature."

   Rule Beta: "If there is nothing anyone can do to change X, and nothing anyone can do to change the fact that Y is a necessary consequence of X, then there is nothing anyone can do to change Y either." This rule has been called the "Transfer of Powerlessness Principle."
6. "There is nothing we can now do to change the fact that our present actions occur" (Kane, *Contemporary* 23-25).

The Consequence Argument explains that if determinism is true, then we cannot have free will because our actions are the necessary consequences of previous events. The Consequence Argument does not necessarily assert whether or not we have free will, but merely explains why free will and determinism are incompatible. We cannot change previous events. We cannot change the fact that our actions are the necessary consequences of previous events. Therefore, we do not have the power to choose to act differently. If our actions are determined, then we cannot have free will.

In order to further defend their perspective, incompatibilists refute compatibilism. Incompatibilists argue that the compatibilist perspective wrongly tells us that agents have the power to do otherwise, when in reality they do not (Kane, *Contemporary* 29-30). If an agent's actions are determined by the laws of nature and by its past experiences, then the agent cannot choose to act differently than how nature and its experiences have led it to act. According to incompatibilist thought, the agent cannot have free will if determinism is true. If the agent's behavior is determined, any influences on the agent determine the decisions the agent makes, eliminating the ability to choose otherwise.

The distinctions within the incompatibilist perspective—libertarianism, determinism, and skepticism—work within the framework of the Consequence Argument to take a stance on whether or not we have free will.

IX. Libertarianism

Libertarianism declares that free will exists because determinism is false. Incompatibilism supports that determinism and free will are incompatible through the Consequence Argument, as explained in the previous section. The challenge for libertarians is to prove that indeterminism is compatible with free will. Because of this question, the free will that libertarians defend is often called indeterminist free will (Kane, *Contemporary* 34). To understand the libertarian perspective, we will acknowledge why some philosophers find indeterminist free will difficult to accept and then investigate how libertarians refute these counterarguments.

Robert Kane details several counterarguments against indeterminist free will in his book. If actions must be undetermined by past experiences, then they must occur by chance (*Contemporary* 34), meaning the agent does not have free will. Chance occurrences cannot be controlled. In the context of human behavior, actions that are taken by chance are similar to impulses or reflexes, neither of which are free. Indeterminist free will would imply that human behavior is unpredictable, impulsive, irresponsible, random, or lucky (35-38). In this case, indeterminism would serve to limit our free will rather than support it.

Furthermore, indeterminist free will implies that the same past experiences could lead to different possible futures. Libertarianism argues that people always have the power to choose to do otherwise. But philosophers who argue against libertarianism point out that this would mean that the "same thoughts, reasoning, beliefs, desires" that would lead a person to make one decision could also lead to a completely different outcome (Kane, *Contemporary* 36). This seems unreasonable. In the case where there is no reason to choose one way over another, the agent may have the "liberty of indifference." But then the choice would be made arbitrarily, in which case the freedom would not represent any power worth having (37). When considering what it would mean for the same past to result in different possible futures, philosophers who argue against libertarianism find either contradictions or freedom that applies to arbitrary choices.

Libertarianism provides several counterarguments to refute these claims. First, libertarians concede that some aspects of human behavior cannot be controlled by our free will. Author Thomas Pink supports the libertarian perspective in his book *Free Will: A Very Short Introduction*. He acknowledges that while human beings can control their actions, "wants and feelings or sensations are passive in the sense of being things that happen to us, rather than being things that immediately arise as our own deliberate doing" (5). Wants and feelings come from within us, but we cannot deliberately choose to change these things. We may not have free will over these aspects of our experience, but we can exercise free will over our actions.

Libertarians explain that actions can be undetermined by past experiences while not occurring by chance. Gottfried Leibniz, an eighteenth-century philosopher, states that previous "reasons or motives" can "incline without necessitating" current choices or actions (Kane, *Contemporary* 36). In this way, our past experiences affect our choices and actions but do not determine them. Leibniz's argument also addresses how the same past experiences can lead to different possible futures. The same past experiences may incline people a certain way, but their individual free wills could lead them to make different decisions.

Libertarians defend indeterminist free will using extra-factor strategies. In order for an agent to have the power to "act or act otherwise, given the same past circumstances and laws of nature," there must be some factor besides the agent's past experiences and the laws of nature that account for the freedom and power to act otherwise (Kane, *Contemporary* 39). Any extra factor justifies the libertarian perspective on indeterminist free will because it explains how actions can be undetermined without being arbitrary or random. We will consider three extra-factor strategies, as explained by Robert Kane: mind-body dualism, noumenal selves, and agent-causation.

Mind-body dualism supposes that there is a distinct separation between mind and body. The body is governed by the laws of nature and affected deterministically by physical events, whereas the mind does not exist in the physical world, and is therefore not deterministically caused. As long as nature allows for some indeterminism—possibly in the brain—the "immaterial mind or soul" could exist without being determined by events or physical laws and could intervene in the physical world (Kane, *Contemporary* 40). The intervention of the mind into the physical world would take the form of human actions. According to dualists, because the mind is undetermined by physical laws, human beings can have free will.

Another extra-factor strategy used to explain indeterminist free will is the noumenal self, as proposed by philosopher Immanuel Kant. Kant believed that “all events occurring in space and time were determined.” However, he also believed that science and reason could only “tell us the way things *appear* to us in space and time,” not “about the way things are in themselves” (Kane, *Contemporary* 43). Therefore, Kant proposed the existence of a noumenal self, which exists outside of space and time, and therefore cannot be explained by science or reason. The supposed contradiction that the same past can lead to different possible futures is only counterintuitive, according to Kant, if choices are explained in terms of “prior states and processes of any kinds, physical or mental.” Because noumenal selves exist outside of space and time, they cannot be explained by previous states (43). The noumenal self is undetermined by the laws that govern the physical world, making it a potential source of our free will. Though it eludes scientific explanation, the noumenal self supports indeterminist free will.

Agent-causation, an idea defended by Roderick Chisholm, is the final extra-factor strategy we will consider. The agent-causal strategy explains that “free agents are capable of causing their own free acts in a special way…that is not reducible to causation by circumstances, events, or states” (Kane, *Contemporary* 44-45). Sciences typically accept direct causation of events by other events. Agent-causation proposes that agents themselves can directly cause events or occurrences. Agent-causes are primitive relations that cannot be described by the typical framework for explaining causes and effects. Richard Taylor, another agent-cause theorist, explains that some “causal chains” can be traced back to a definite beginning, and often these beginnings occur “with the agents themselves” (46). Agent-causation cannot be explained by the familiar language for describing chains of causes and effects, but nevertheless provides an extra factor that justifies indeterminist free will.

There are some logical problems with these extra-factor strategies proposed by libertarians. The extra factors are mysterious, obscure, difficult to explain in terms of familiar terminology, and often transfer the problems with indeterminist free will presented at the beginning of the section to some immaterial entity (Kane, *Contemporary* 47). Even Kant, despite his belief in the noumenal self as a reasonable explanation for libertarian free will, agreed that libertarian free will is necessarily mysterious and difficult to understand (44). Because of these problems, philosophers who argue against libertarianism remain unsatisfied with extra-factor strategies to justify the existence of free will.

Libertarians also defend free will against the advances in the social sciences that seem to show that human behavior is deterministically caused by our past experiences. In response to such scientific research, Pink argues:

> No one has actually shown that determinism holds at the level of human action. Our actions are often predictable. Yet these predictions generally fall short of certainty. We find tendencies that many human actions follow. But these do seem to be tendencies only, not iron laws, and individual actions can still break the pattern. Belief in the wide-scale predetermination of human actions remains no more than a guess or speculation—a speculation that as yet remains not even probable, still less proved. (15-16)

Pink suggests that while progress in some fields of science may yield results that seem to support deterministic human behavior, there are still exceptions. Furthermore, research in chaos, which shows unpredictable and counterintuitive behaviors even in seemingly simple systems, has been used as evidence of the indeterminism that libertarians rely on for free will (Kane, *Contemporary* 134). Human beings represent complex systems. While there may be speculations as to which behaviors will be exhibited and the causes of these behaviors, there are always individuals that cannot be explained by these tendencies. For these reasons, libertarians argue that human behavior is not determined by our past experiences and we have the power to exercise free will.

X. Determinism

Determinists believe that the world is deterministic, meaning free will is impossible. In his popular book *Free Will*, Sam Harris explains his determinist perspective in a way that is accessible to readers. Harris states, "Free will *is* an illusion. Our wills are simply not of our own making. […] We do not have the freedom we think we have" (5). He argues that human behavior is the result of "cause-and-effect relationships" (11). Harris cites several studies conducted as recently as 2011 that show that brain activity that will develop into a particular action can be detected before the human is consciously aware of having made a decision. In the time it takes for the brain activity to manifest itself into an action, we get the sense that we "have complete freedom to behave however you please," but in reality, "your brain has already determined what you will do" (9). Our behavior is deterministic. We cannot control the factors that form our thoughts and actions. Underlying causes that we may not even be aware of—such as our genes, our past experiences, or the wiring of our brains—determine our behavior.

Harris proceeds to refute a counterargument to the determinist perspective. Daniel Dennett, a compatibilist and Harris's friend, argues that we can be unaware of the causes of our behavior and be free:

> Anything that our brains do or decide, whether consciously or not, is something that *we* have done or decided. The fact that we cannot always be subjectively aware of the causes of our actions does not negate free will—because our unconscious neurophysiology is just as much "us" as our conscious thoughts are. (20)

Dennett and others who might argue against Harris on this point, believe that even if we are not aware of all of the underlying factors that contribute to our thoughts, decisions, or actions, these choices are still ours because they come from our own brains. We are in control of our decisions and actions—even if we are not consciously aware of how we are exerting this control—therefore, we are free. To argue against this, Harris explains that we do not claim responsibility for the "decisions" that other organs in our bodies are making. He asks, "Are *you* producing red blood cells and digestive enzymes at this moment?" According to Harris, it is absurd to take responsibility "for everything that goes on inside your skin because it's all 'you'" (23). We are not responsible for the "decisions" our body makes. Sometimes—for example, in the case of unforeseen medical problems—we are a victim of the workings of our organs. While our genes, experiences, and brains are the underlying causes that determine our decisions, we are not responsible for these causes nor the resulting decisions. We are not free.

Another issue with this counterargument against determinism is that even if we could take responsibility for the work of our unconscious brains, we still would not be free. The perspective of Dennett and similar thinkers claims that we can consider ourselves to be free because our own brains are making decisions for us, even if we are not conscious of the underlying causes of these decisions. However, Harris explains that claiming "that 'my brain' decided to think or act in a particular way, whether consciously or not, and that this is the basis for my freedom, is to ignore the very source of our belief in free will: the feeling of *conscious* agency" (26). The basis of

Harris's deterministic perspective is that our thoughts and actions "cannot be traced to a point of origin in our conscious minds" (6). Our intuition justifies our free will because we feel like we are in control and we are consciously making decisions for ourselves. Even if we could take responsibility for the decisions our brains make for us, we would not be free. The factors that influence our decisions and actions cannot be traced to causes within our conscious control, which is the basis of our feelings of free will.

Determinism and the realization that we are not free comes with a number of important consequences. One such consequence that Harris acknowledges is the importance of freedom. Harris states that our lack of personal free will "does not make social and political freedom any less important. The freedom to do what one intends, and not to do otherwise, is no less valuable than it ever was" (13). Though we do not have personal free will, freedom in social and political contexts is still important. People should have the freedom to think and act in the way they choose without fearing coercion to think or act another way. Therefore, it is not contradictory to acknowledge a lack of personal free will while fighting for freedom within political systems and society as a whole.

Another consequence of determinism is the importance of choice. Some people may interpret the fact that the choices we make are not under our control to mean that our choices are insignificant. However, Harris argues that human choice is a powerful force regardless of the causes of the choices. Harris writes:

> And the fact that our choices depend on prior causes does not mean that they don't matter. […] Decisions, intentions, efforts, goals, willpower, etc., are causal states of the brain, leading to specific behaviors, and behaviors lead to outcomes in the world. Human choice, therefore, is as important as fanciers of free will believe.

> But the next choice you make will come out of the darkness of prior causes that you, the conscious witness of your experience, did not bring into being. (34)

The power and significance of our choices is not diminished by the fact that we do not have the free will to consciously make these choices. We may not be in control of the factors that cause our thoughts and actions, but our choices still matter. Human choice is a force that leads to outcomes that can effect change in the world. As a result, our decisions have significance determined by the consequences of our choices in the world around us.

Responsibility for our actions is called into question by the determinist perspective on free will. Harris begins his book by detailing the gruesome attack of the Petit family—William, Jennifer, and their daughters, Hayley and Michaela—and murder of most of the family—William escaped and survived—by Steven Hayes and Joshua Komisarjevsky (1-3). Harris appeals to the reader's general feeling that Hayes and Komisarjevsky should be held responsible for their actions in one way or another. However, Harris points out that holding the men responsible does not make sense, because they have no way of understanding why they are the way they are. They have no free will with which to choose how they behave. Harris points out that if he could become Hayes or Komisarjevsky, "if I had his genes and life experience and an identical brain (or soul) in an identical state—I would have acted exactly as he did" (4). When we consider our lack of free will, "even the most terrifying sociopaths begin to seem like victims themselves. The moment we catch sight of the stream of causes that precede their conscious decisions, reaching back into childhood and beyond, their culpability begins to disappear" (18). The actions of the men towards the Petit family are no less horrifying, but ultimately they are not responsible for their actions. Their actions were deterministically caused by their genes and previous experiences.

Our lack of responsibility for our own intentions and actions calls the ethics of our legal justice system into question. The legal justice system depends on the idea that people deliberately choose to act a certain way, and as a result are responsible for their actions. But if our actions are the result of a "mental cause and effect," can we hold people responsible and deal justice in the same way? Harris reflects on weather predictions as an example of complex but deterministic behavior. Weather is a chaotic system, meaning that while short term predictions might be useful, long term predictions are unreliable because of the complexity of the system. Similarly, human behavior is deterministic, meaning that knowing about someone's genes or past experiences could give us an idea about how they might think or behave. However, the brain is sufficiently complicated to make such predictions unreliable; we cannot fully understand the factors that cause a person's thoughts and actions. Harris asks, "If we view people as neuronal weather patterns, how can we coherently speak about right and wrong or good and evil?" (48) Harris does not propose answers to these questions. He leaves the reader to consider how our inability to comprehend the causes of human behavior and our lack of free will—and ultimately, responsibility for our actions—could affect the ethics of our legal justice system.

Harris empowers readers to use the knowledge that we do not possess free will. Though we may not be in control of determining what we do, we can still change. When we deliberately try to make a change to improve ourselves, our capacity for change is determined by what we have learned from our past experiences (39). In the end, "you will do whatever it is you do" (44). "Am I free to change my mind? Of course not. It can only change *me*" (65). By impacting "the world around us and the world within us," we contribute to the influences that affect other individuals (62). This means that regardless of whether or not we have free will, our choices and actions are significant and can become powerful forces for change. "You are not controlling the storm, and you are not lost in it. You *are* the storm" (14). We are participating in a storm of influences that affect our experiences and perceptions. We cannot control these influences, but we need not feel helpless. Just as much as we are affected by others' thoughts and actions, our own thoughts and actions feed back into the storm to affect the world around us. Though we do not have the freedom to determine how we act or how our actions impact others, we are nevertheless contributing to the change in other individuals and in the world.

XI. Skepticism

Sources on the free will debate generally acknowledge the existence of skeptics within the incompatibilist perspective, however the skeptics' stance on questions of free will is much broader and less unified than other perspectives. For this reason, the nuances of skepticism will not be discussed at length. However, for a complete understanding of the complexity of the free will debate and the scope of perspectives on the issue, we will briefly overview some views within skepticism.

One skeptic perspective explains that free will is inconsistent with both determinism and indeterminism. According to skeptics, if determinism is true, our thoughts and actions are determined by our past experiences, meaning that we do not have the power to control or influence our own actions. If our actions are undetermined, then they occur randomly or by chance, meaning, again, that we do not have the power to exert our free will on our own actions (Harris 5; Pink 16). According to some skeptics, whether or not determinism is true is irrelevant because free will cannot exist in either case.

Skepticism is used to identify philosophers who are skeptical about the existence of free will. For example, philosophers that identify as hard determinists—a perspective that qualifies some of the arguments made in the determinist perspective, but nevertheless agrees that free will is an illusion—are considered skeptics, as Kane explains in his book, *A Contemporary Introduction to Free Will*. Philosophers seem to use skepticism as a general term for alternative views on the free will debate that, based on the fine details and qualifications of their beliefs, cannot be clearly identified as libertarian or determinist. Because of the lack of consistency in defining skepticism, skepticism is not the focus of this paper. However, the particular skeptic perspective addressed in this section seemed well defended across the literature. Skepticism may not play a major role in the project, but it does provide a more complete understanding of the variety of incompatibilist perspectives on free will.

XII. Compatibilism

Compatibilism is the belief that determinism and free will are compatible. Though our actions are determined by our past experiences, we are still free to make our own choices and to act otherwise. Compatibilists, as explained by Robert Kane, defend their perspective by challenging the definition of free will and misunderstandings about determinism.

Compatibilists defend an alternate definition of free will than the one addressed within the incompatibilist perspectives. Compatibilists do not believe in the "'deeper' sense of free will" that involves "ultimate control over what you will or want." This definition of "deep" free will has been the definition used to discuss the incompatibilist perspectives in previous sections of this paper. According to compatibilists, "deep" free will is impossible because our actions are determined by our past experiences, over which we have no control. "Deep" free will is incompatible with determinism. Instead, compatibilists define free will as "the ability to choose or decide as you will without constraint" (Kane, *Contemporary* 15-16). Compatibilists argue that the freedom of action—acting or acting otherwise, if the agent wants to do so—and freedom of will—choosing or deciding without constraint or coercion—are more important than the impossible "deep" free will that incompatibilists discuss. Using this definition of freedom, the compatibilist perspective argues that free will is compatible with determinism.

Compatibilists must explain how, unlike in the incompatibilist perspectives, determinism and free will are compatible. Indeterminism is incompatible with freedom because it requires that the same past experiences could result in different possible futures. This means that "the same prior deliberation, the same thought processes, the same beliefs, desires, and other motives" that lead a person to make one decision could lead a person to a completely different outcome (Kane, *Contemporary* 16). Rather than implying greater freedom, incompatibilism seems to bring a lack of freedom or arbitrary freedom. On the other hand, compatibilists argue that determinism can be shown to coexist with free will. Determinism requires that the same past experiences will lead to the same future. The compatibilist analysis of actions focuses on determining whether or not the agent had the freedom to act otherwise if they had wanted to. For an agent to have the freedom to do otherwise, their past experiences would have to incline them towards a different decision or action than the one they made. In other words, the agent "would have done otherwise, if things had been different—if the past had been different in some way" (16). The slightest change in a person's past experiences would completely change the influences determining their present actions. However, this person still has the freedom to do and act as they choose so long as there are no constraints on their freedom. And this person has the power to do and act otherwise if their thoughts and actions had been determined in a different way. Therefore, compatibilists say, free will is compatible with determinism.

In order to further defend the relationship between determinism and freedom, the compatibilist perspective challenges common preconceptions and misunderstandings about determinism. The first misunderstanding is that determinism implies constraint, coercion, or compulsion. Freedom is the opposite of constraint, coercion, and compulsion because these concepts fight against our ability to do what we want. However, freedom is not the opposite of determinism because the influences that determine our actions—such as the laws of nature, our past experiences, or our character—do not prevent us from doing what we want to do. A. J. Ayer, a twentieth-century compatibilist, explained the source of this common misconception about determinism:

> Many people think freedom is inconsistent with determinism because they have a mistaken image of natural causes or laws of mature “overmastering” us, forcing us against our wills. But, in fact, the existence of laws of nature indicates only that certain events follow others according to regular patterns. To be governed by laws of nature is not to be in chains. (Kane, *Contemporary* 18)

The influences that affect our thoughts and actions do not limit our freedom. Though we may not have control over these influences, they simply represent patterns in our own behavior. Because determinism does not constrain the actions we wish to take, determinism does not conflict with our freedom.

Another misconception about determinism is that causation implies constraint. Constraints may be causes that determine certain behaviors which are not performed out of our own free will. But the lack of freedom is due to constraint, not causation. Compatibilist David Hume explains that our actions are caused by our character. In this sense, our actions are both determined and the result of our freedom. If an action was performed out of character or under some sort of constraint, we could not be held responsible for that action because we did not perform it willingly (Kane, *Contemporary* 18-19). Though our past experiences and our character work to determine our actions, the freedom we have in performing those actions is not minimized. In fact, this is the source of our free will: we have the power to think and act as we choose because of our characters and past experiences.

Determinism should not be confused with control by other agents. Compatibilist Daniel Dennett explains that we are averse to being controlled by other agents because this control diminishes our own freedom. However, the laws of nature that determine our actions do not control us because nature is not an agent (Kane, *Contemporary* 19). Determinism does not represent a form of control over our thoughts and actions. Therefore, it is compatible with our freedom.

Determinism is often confused for fatalism. Fatalism is the idea that "whatever is going to happen, is going to happen, *no matter what we do*" (Kane, *Contemporary* 19). Determinism, however, is not synonymous with fatalism, as explained by compatibilist John Stuart Mill. Fatalists believe that because events are deterministically caused, "there is no use in struggling against it; that it will happen however we may strive to prevent it" (19). For example, fatalists would assume that a person's character is formed *for* them. But Mill states that while a person's "character is formed by [their] circumstances…[their] desire to mold it in a particular way is one of those circumstances, and by no means the least influential" (20). The choices we make of our own free will affect our circumstances, which in turn influence our character and our future thoughts and actions. In this way, we see that future events are in no way inevitable. Even if our choices are determined and influenced by other factors, our freedom of will and freedom of action are not trivial. We have the freedom to choose, and the outcome of these choices will determine our future decisions and even future events.

Determinism is often compared with mechanism. Some people fear that determinism would imply that our actions would be mechanical, automatic, or fixed responses to our environment, such as those of computers or insects. However, compatibilists point out that determinism cannot be equated to mechanism because humans are significantly different from machines and insects. We have moods and feelings, "we reason and deliberate, question our motives, reflect on our values, make plans about the future, reform our characters, and make promises to others that we then feel obligated to keep" (Kane, *Contemporary* 20-21). Because machines and insects are incapable of these things, our behavior is quite distinct from mechanical or automatic behaviors. Furthermore, determinism does not diminish our capacity for these complex behaviors. Even if our behaviors are determined, we are nevertheless free to question, reflect, and change as we want. Based on the refined understanding of determinism and its implications, compatibilists argue that determinism is not only compatible with free will, but necessary for freedom.

XIII. Evaluation of the Perspectives on the Free Will Debate

In order to understand the complexity of philosophical questions of free will and determinism, it is important to be familiar with the variety of stances on the free will debate, including the incompatibilist perspectives—focusing on libertarianism and determinism—and the compatibilist perspectives. However, before we can integrate the mathematics and philosophy of chaos, we will briefly evaluate each of the views on the free will debate in the context of this project.

Libertarianism relies on extra-factor strategies. Because the extra factors that would explain indeterminist free will are immaterial, we cannot discuss them using familiar terminology used to describe cause and effect relationships or interactions in the physical world. In fact, the lack of language to adequately define these factors or explain their role in the world make them difficult to defend. Determinism is better defended than libertarianism. Determinists can even draw on specific scientific studies to support their claims. However, as an incompatibilist perspective, determinism addresses “deep” free will, which does not necessarily affect how we think about our actions in our everyday lives. For these reasons, the incompatibilist perspectives will not be the focus of the remainder of the paper.

The compatibilist perspective relies on determinism and an alternate definition of free will. Determinism agrees with progress made in the physical and social sciences as well as the discussion of chaotic systems in section introducing chaos theory at the beginning of the paper. Furthermore, the alternate definition of freedom of will and freedom of action is more practical and applicable than “deep” free will. The compatibilist definition of free will affects how we think about and understand our freedom in our everyday lives. For these reasons, the remainder of the paper will focus on the compatibilist perspective on the free will debate.

XIV. Integration of the Mathematics and Philosophy of Chaos

With a working understanding of topics in the mathematics of chaos and philosophical questions of free will, we can make connections between these two disciplines. From the mathematics and philosophy of chaos, we will argue that we are responsible for our contributions to our surroundings because our thoughts and actions are forces for unpredictable change.

We must first consider how these two disciplines can relate to one another. Following the compatibilist perspective, determinism and free will are compatible, meaning that although our actions are influenced by our past experiences, we have free will. Determinism is one of the characteristics of chaotic systems. Furthermore, free will seeds chaos (Smith 4). In addition, “all difficult decisions are made under uncertainty” (160). The science of chaos works with uncertainty in systems. Chaos can allow us to cope with uncertainty when making decisions and understand that our choices could have unpredictable consequences.

Responsibility for our actions and free will are closely related concepts. In fact, this connection is so important that philosopher Robert Kane has proposed an alternate definition of free will in terms of responsibility. According to Kane, any decision or action for which we are directly responsible must have been performed of our own free will (Kane, *Contemporary* 80). Considering this alternate definition of free will and the fact that we have free will according to the compatibilist perspective, we can conclude that our free will brings with it a responsibility for our actions.

We can consider society or the world as a whole to be chaotic systems. By assumption of the compatibilist perspective on free will, the world is deterministic because it is governed by laws of nature. Additionally, if we consider an event, we can usually trace a chain of causes that led to that event. When agents get involved, it may be more difficult to distinguish the causes that led to this event because behavior is determined by factors we may not be aware of. However, these causes still exist, making the world deterministic. The world is also sensitively dependent because small changes could lead to outcomes that quickly diverge until they proceed independently. It is nonlinear because certain actions often have disproportionate effects. To see that these characteristics hold, we can consider examples of decisions that felt insignificant at the time, but ultimately changed the course of our lives. If we had not made that one seemingly insignificant decision, our lives would be completely different. This idea that small changes can lead to disproportionately large effects in the long term shows that the world is sensitively dependent and behaves nonlinearly. Because the world has all three characteristics of chaotic systems, we can conclude that the world is a chaotic system.

Our actions serve as inputs to the chaotic system of the world. This means that the consequences of our actions are unpredictable. “In a chaotic system, everything is connected, through negative and positive feedback, to everything else” (Briggs and Peat 34). Even the smallest action can create feedback. In the discussion of the Feigenbaum attractor, we saw that small changes in parameter values changed the long term behavior of the system. The feedback from a single action can have immense effects. “Our attitude and being forms the climate others live in, the atmosphere they breathe. We help supply the nutrients for the soil where others grow” (41). Our choices could affect the people around us, who take actions that effect the people around them, and so on, until our initial choice is fed back throughout the system, affecting more people than we could have imagined. Enough small changes can create sufficient feedback to bring the system to a bifurcation point that completely alters the behavior of the system. Our choices interact with and affect one another, meaning that any moment has the potential for immense change. Because the world is chaotic, we may not be able to predict the consequences of our actions. However, we can see that our choices matter because they affect our surroundings, including people, their decisions, and future events. Our actions do make a difference.

Even if we do not have free will, within the context of the chaotic world, our actions can still effect change. Both the determinist and compatibilist perspectives on free will explain that determinism is true, which allows us to consider the world as a chaotic system. As long as determinism exists, our characters and choices will have nonlinear effects. "There is no telling how much I might change in the future. […] A creative change of inputs to the system—learning new skills, forming new relationships, adopting new habits of attention—may radically transform one's life" (Harris 46). Our choices give us the power to change the world on some scale. Furthermore, others' decisions will contribute to the factors that affect us, meaning we have the capacity to change as well. Every decision has some effect that serves as an input to the chaotic system of the world. This input contributes to the chain of causes and effects that influence others. A single action creates change that can be felt on some scale greater than the individual.

Despite our chaotic free will, patterns emerge in the form of society or other groups. The complexity of human behavior produces interesting bands of order within the chaos of our individual free wills. This idea relates to the bands of order that emerged out of chaotic behavior in the Feigenbaum attractor. We can also make a connection to fractals, which balance chaos with structure. In their book *Seven Life Lessons of Chaos*, which combines philosophical and spiritual thought with the lessons of chaos theory, authors John Briggs and F. David Peat explain:

> Each of us as an individual is inter-connected to the systems of nature, society, and thought that surround and flow through us. We live within movements constantly affecting each other and creating an unpredictable chaos at many levels. Yet within this same chaos is born all the physical and psychological order that we know. (4)

Our individual free wills seed chaos in the world. Yet in the midst of all of this chaos, society is formed. We are closely tied together by the structure that emerges out of the chaotic feedback loops created by human behavior. We can exercise our free will to create change on some scale—be it one person, a local community, a nation, or even the whole world—because of our interconnection. The reach of our actions does not necessarily matter. Whatever we contribute to the world—which will be fed back through the structures and patterns that form as a result of our chaotic free will—can affect those around us as well as ourselves, creating change.

Philosophical thought on free will and determinism allows us to consider the origins of our own thoughts and actions and our responsibility for our behavior. Chaos theory teaches us about change, feedback, and the relationships between order and disorder, simplicity and complexity. Taken together, we recognize how chaotic the world is. Our actions are influenced by our character and determined by our past experiences and the laws of nature. Our freedom of will and freedom of action contribute to the chaos. However, within the chaos of our individual freedoms, patterns emerge. Every action is an input to the chaotic system of the world. An action feeds back through the system, affecting the factors that determine others' decisions and certain events. We are responsible for our choices because they have consequences. We may not be able to predict the results of our actions. But considering philosophical questions of free will in the context of chaos theory makes it apparent that our choices will affect the world we live in.

XV. Conclusion

The mathematics of chaos theory and philosophical questions of free will show us we must take responsibility for our actions because of their power to change the world in unforeseen ways. We began our journey by exploring topics of chaos theory through mathematical dynamical systems and computer simulations, such as the Feigenbaum attractor and the fractal Sierpinski triangle. We then considered the nuances of the free will debate by examining incompatibilist—libertarian and determinist—and compatibilist perspectives. For the purposes of this project, the compatibilist perspective proposed the strongest arguments. Focusing on the compatibilist perspective on free will and considering the characteristics of chaotic systems, we reasoned that our free will comes with immense responsibility for our actions. Our choices, no matter how small, feed back into the chaotic system of the world and contribute to its long term behavior on some scale.

Chaos theory provides further justification for compatibilist perspective on free will. Philosophical perspectives allow us to contextualize the lessons from chaos theory in our everyday lives. We have the freedom, responsibility, and power to influence someone’s decisions, start a movement, or change our surroundings. We must be willing to make the most of our free will and take responsibility for our actions in order to make the most worthwhile contribution to the chaotic system of the world.

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Appendix A

Quasiperiodic Motion

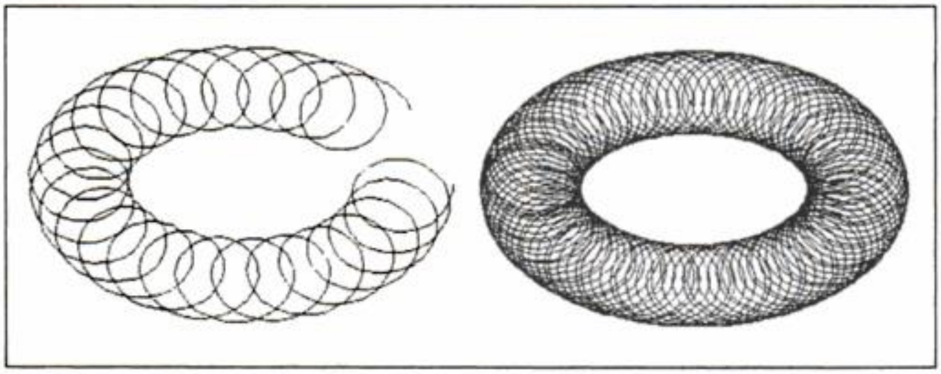
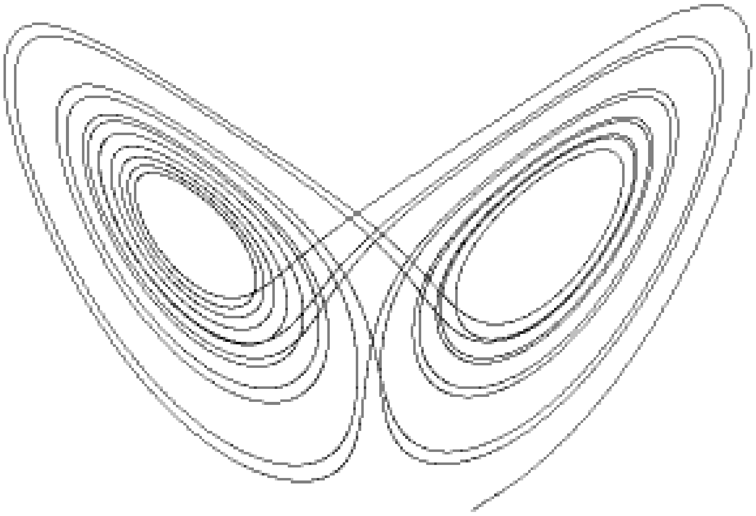


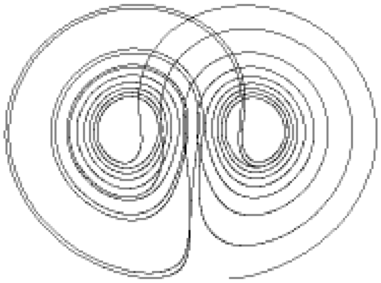
Figure 4.2 The combination of two independent periodic motions produces a torus. A quasiperiodic orbit winds itself around the torus like thread on a spool.

This image, from Ian Stewart's article "Portraits of Chaos," shows an example of quasiperiodic motion. Quasiperiodic motion combines two or more periodic orbits. The resulting donut shape is referred to as a multidimensional torus.

Appendix B

Lorenz Attractor

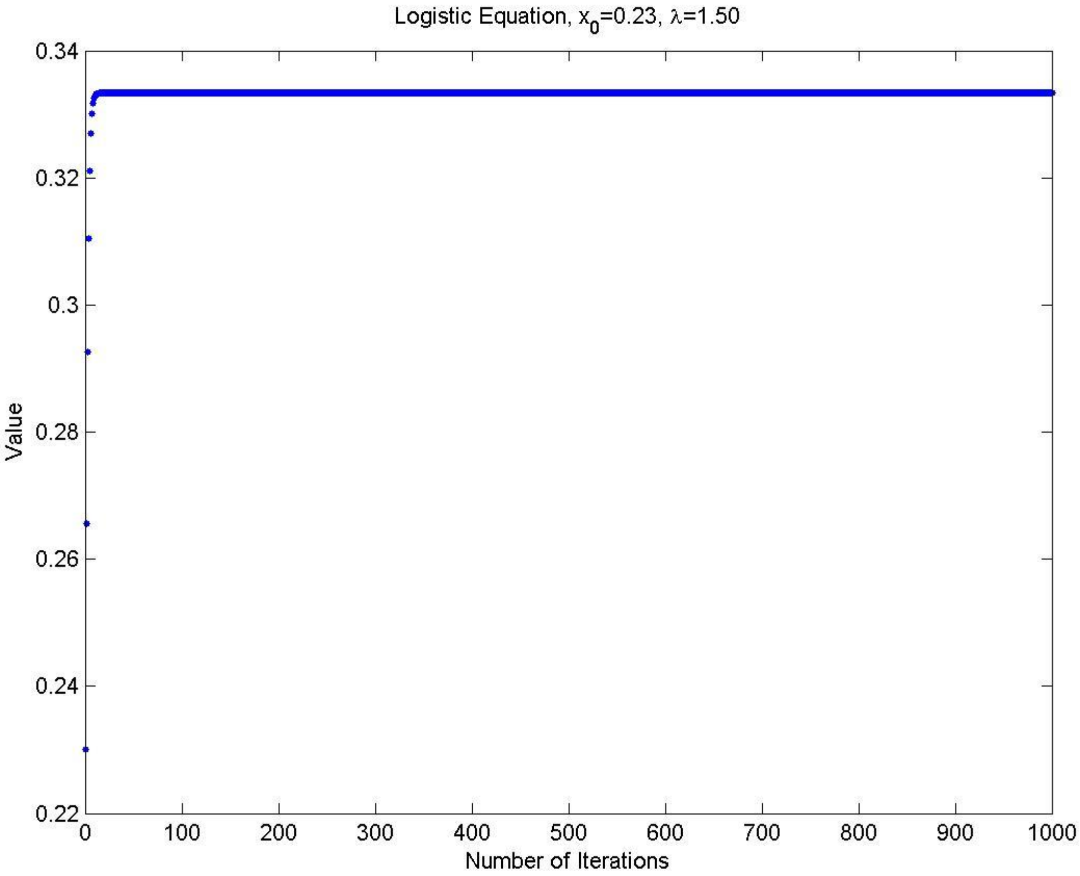




These images, from Larry Bradley's article "Strange Attractors," offer different perspectives on the three-dimensional Lorenz attractor. Its butterfly shape has become tied to the butterfly effect—the idea that small changes can make big differences. The Lorenz attractor is one image that has come to represent chaos.

Appendix C

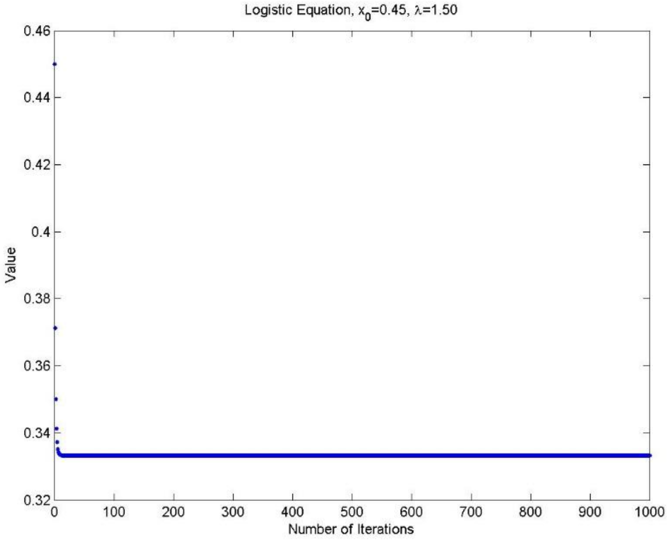
Long Term Behavior of the Logistic Equation

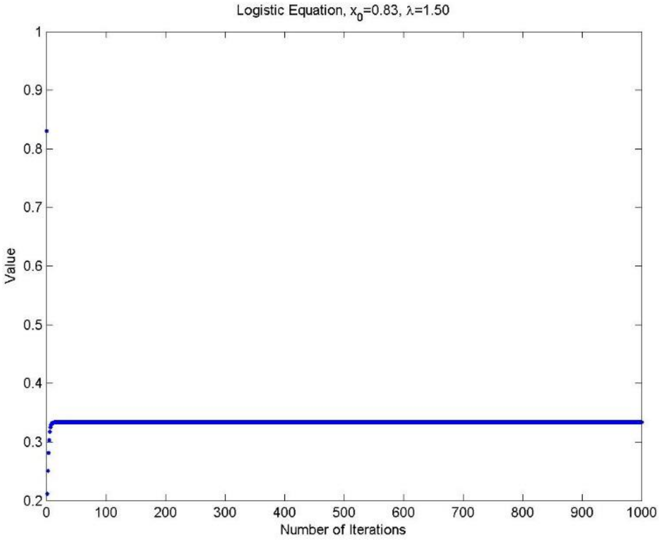


The above graph shows 1,000 iterations of the logistic equation with $x_0 = 0.23$ and $\lambda = 1.50$. In this case, the system has a fixed point attractor around 0.33.

Appendix D

Comparing Long Term Behavior of the Logistic Equation with Different Initial Values

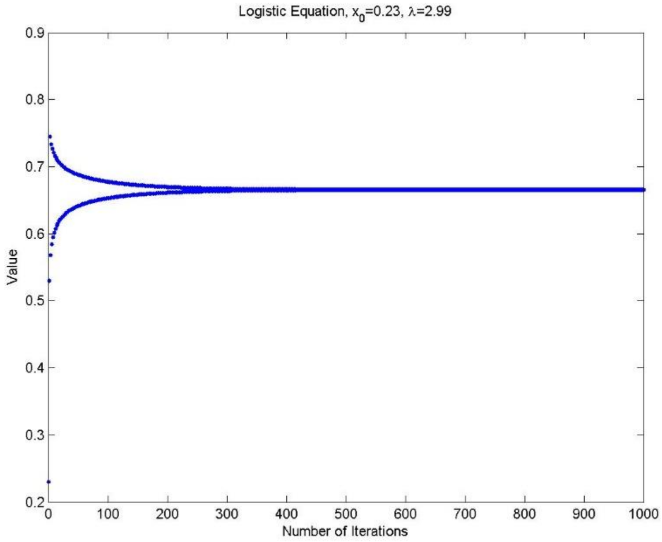


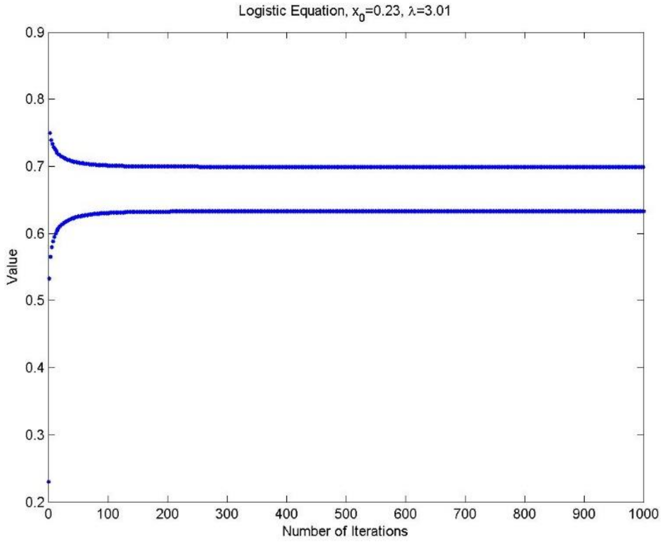


Both of these graphs show 1,000 iterations of the logistic equation with $\lambda = 1.50$. The top graph has $x_0 = 0.45$ while the bottom graph has $x_0 = 0.83$. Both systems tend toward a fixed point attractor at approximately 0.33 (note that different scales along the y-axis). The initial conditions of the logistic equation do not affect the attractor for the particular value of λ.

Appendix E

Period Doubling Bifurcations in the Logistic Equation

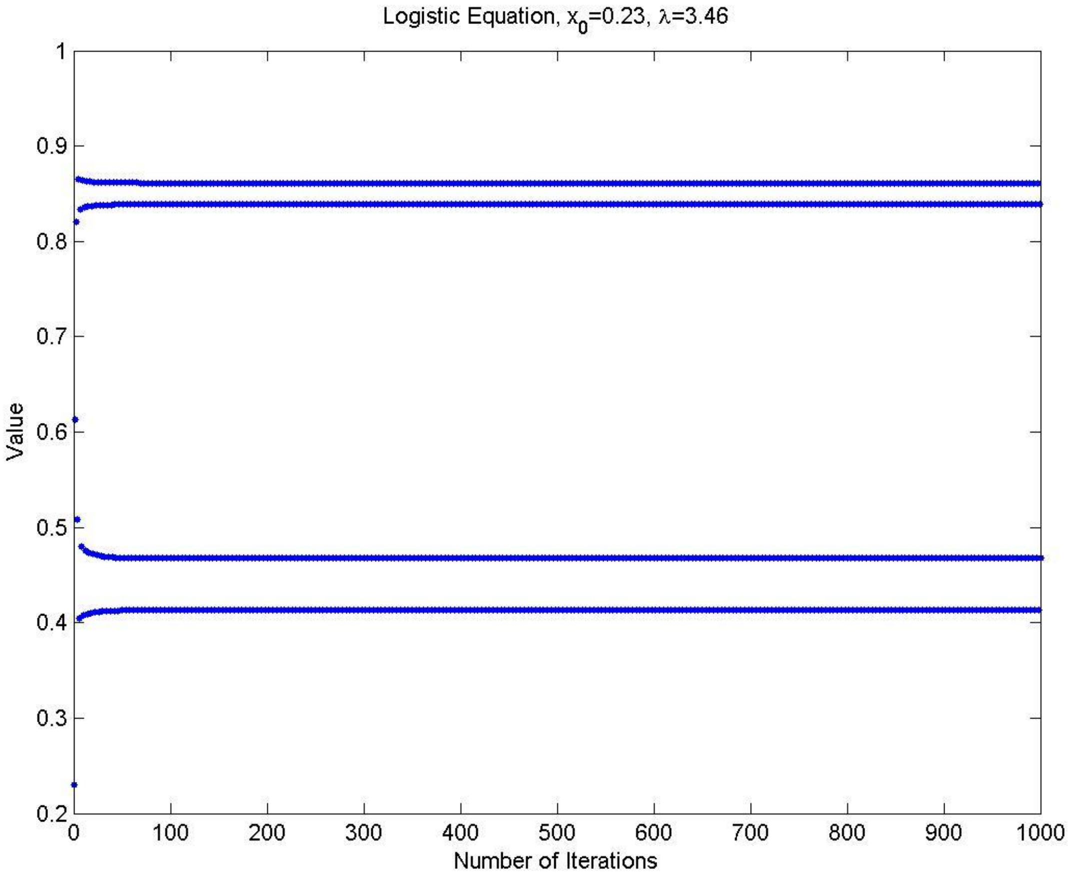




Both of these graphs show 1,000 iterations of the logistic equation with $x_0 = 0.23$. The top graph has $\lambda = 2.99$ and a fixed point attractor. The bottom graph has $\lambda = 3.01$ and a periodic attractor of period two. Based on these graphs, $\lambda = 3.00$ represents a bifurcation point at which the period of the attractor doubles.

Appendix F

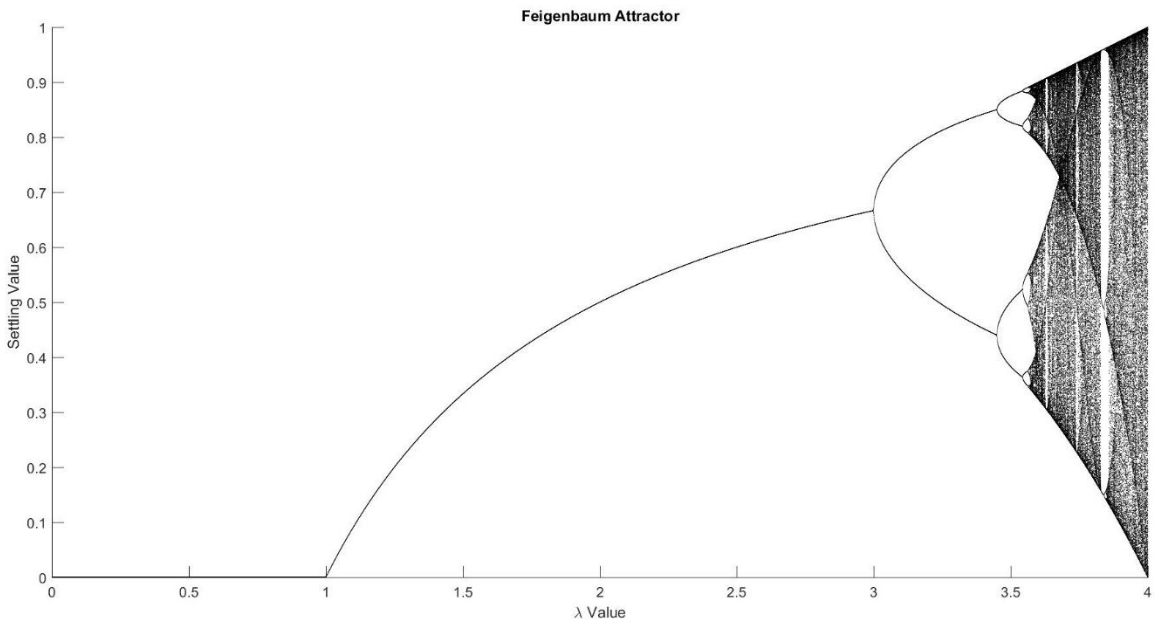
Periodic Attractor of Period Four in the Logistic Equation



This graph shows 1,000 iterations of the logistic equation with $x_0 = 0.23$ and $\lambda = 3.46$. We can see that the long term behavior of this system periodically orbits between four distinct values. We have found another bifurcation point at which the period of the attractor doubles.

Appendix G

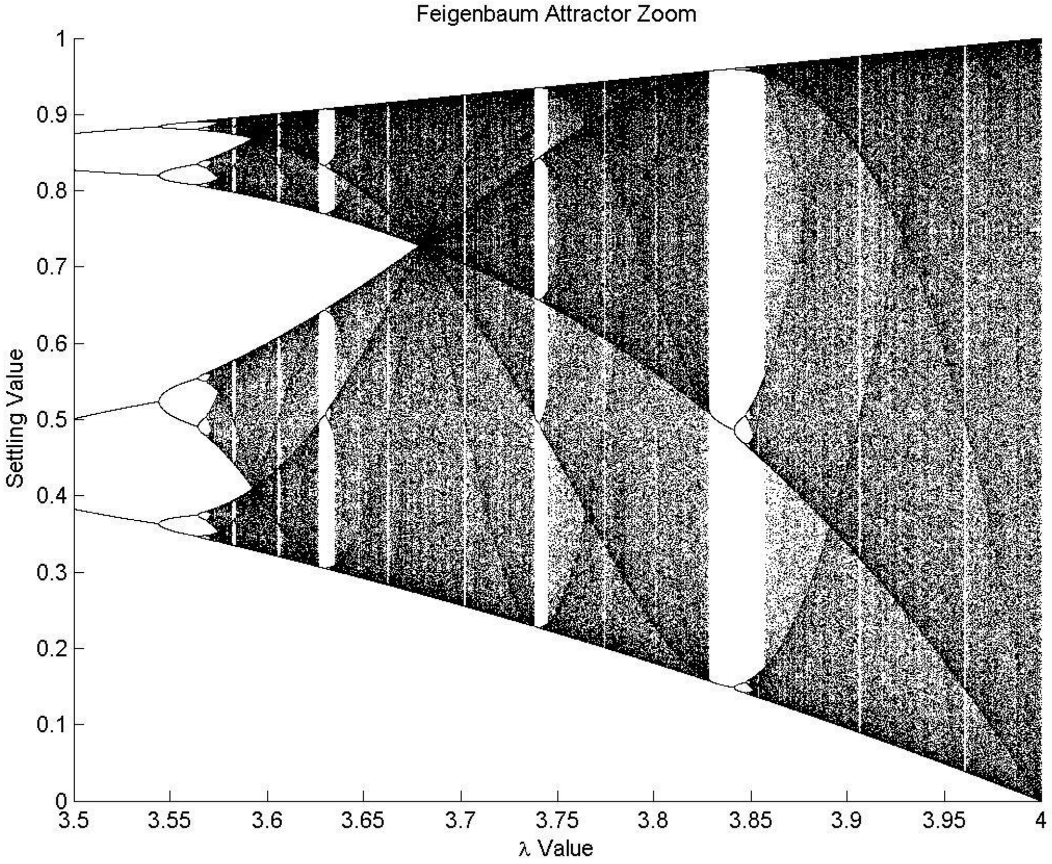
Feigenbaum Attractor



This image is the result of a computer simulation in MATLAB and depicts the Feigenbaum attractor with consecutive λ values differing by 0.0005. When λ = 4.00, almost 200 distinct settling values are plotted, representing chaotic behavior of the system. Note that λ is not allowed to be greater than 4.00. When λ = 4.01, for example, the system quickly approaches infinity, and in less than 30 iterations of the logistic equation, the state values have gotten too big for the computer to compute. Because the parameter λ can be interpreted as a quantification of the feedback, or the amount to which the old *x* value affects the new *x* value, any λ > 4.00 can be interpreted as allowing for too much feedback in the system. For this reason, images of the Feigenbaum attractor only represent the system when λ ≤ 4.00.

Appendix H

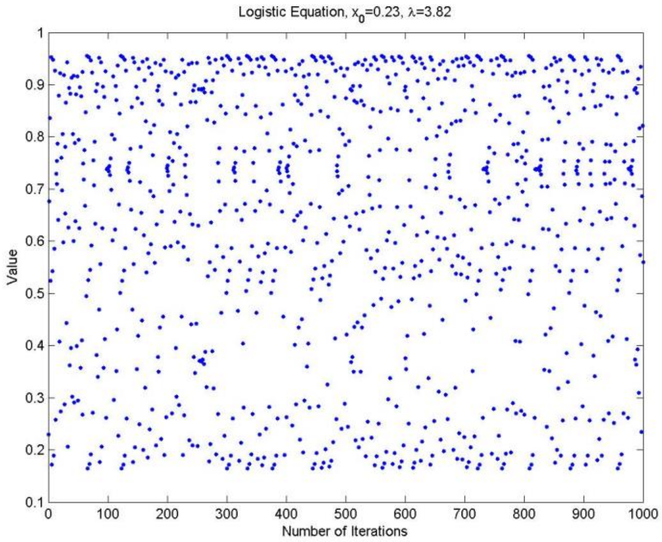
Zoomed in Feigenbaum Attractor

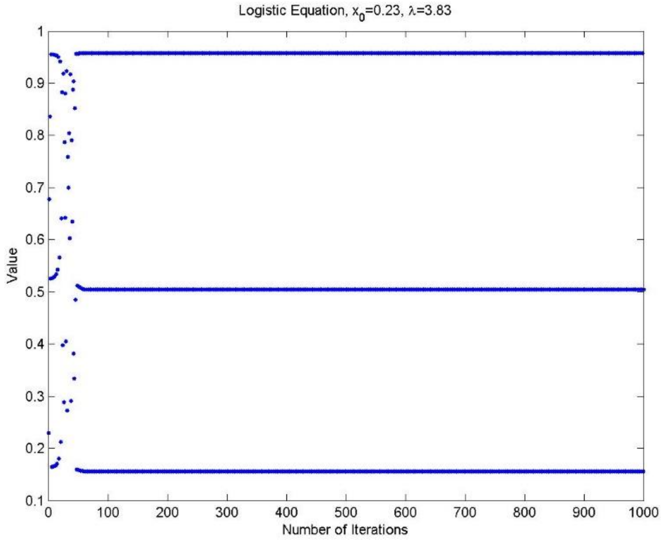


This image is a subset of the Feigenbaum attractor with λ values between 3.50 and 4.00. This image was the result of a computer simulation in MATLAB with consecutive λ values differing by 0.0001. This view of the Feigenbaum attractor allows us to see the period doubling bifurcations more clearly. We can see transitions from period four to period eight to period sixteen. More importantly, we can see that within the chaotic behavior of the system, there is a band of order in which the system settles into a period three orbit. The periods then begin to double before devolving into chaos once again.

Appendix I

Chaotic Behavior to Period Three Orbit in the Logistic Equation

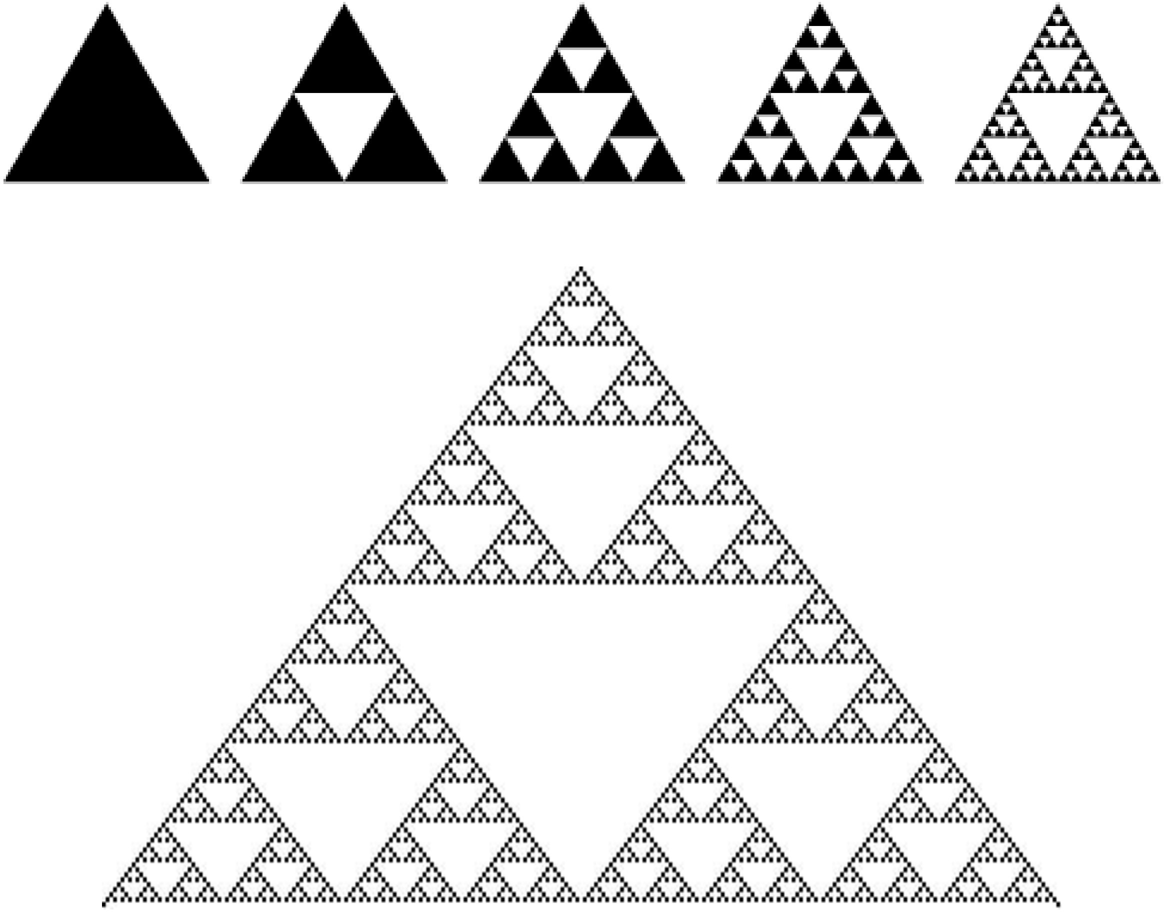




Both of these graphs show 1,000 iterations of the logistic equation with $x_0 = 0.23$. The top graph has $\lambda = 3.82$ and shows chaotic behavior. The bottom graph has $\lambda = 3.83$ and a periodic attractor of period three. Based on these graphs, $\lambda = 3.83$ represents the beginning of a band of order within the chaos of the Feigenbaum attractor.

Appendix J

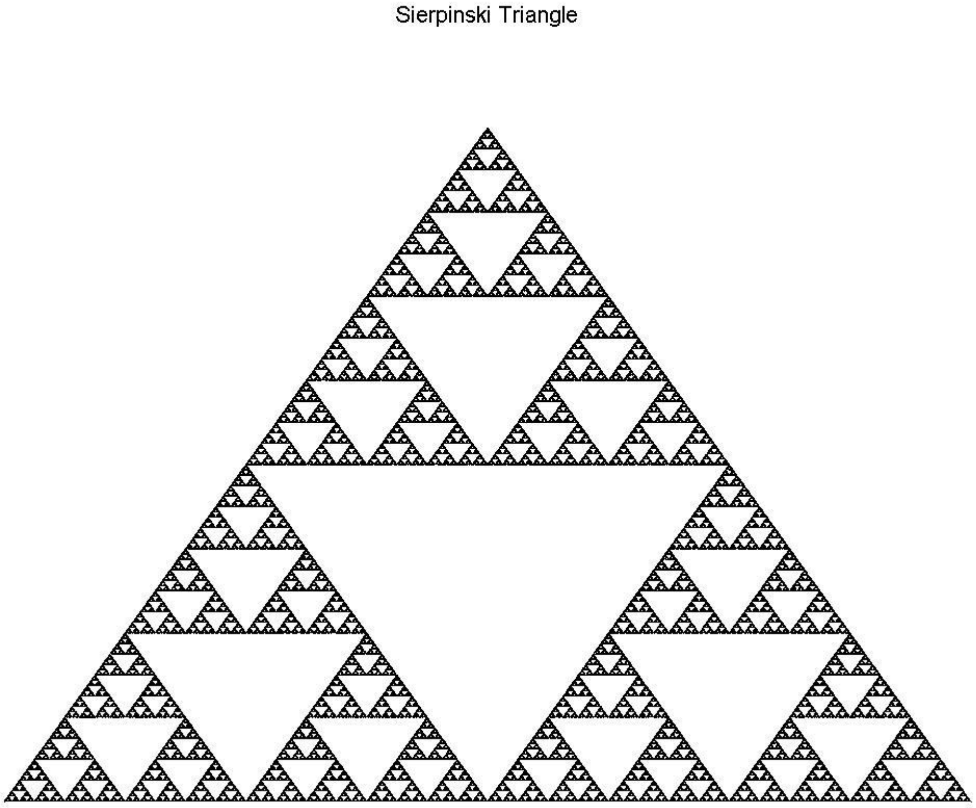
Sierpinski Triangle



These images, from Eric W. Weisstein’s article “Sierpiński Sieve,” show the formation of the Sierpinski triangle as repetition of a rule—removal of one quarter of the triangle’s area. One of the properties of the Sierpinski triangle is self-similarity, a characteristic of mathematical fractals.

Appendix K

Sierpinski Triangle Computer Simulation



This image is the result of a computer simulation in MATLAB and depicts the Sierpinski triangle. Rather than the conceptual creation of the Sierpinski triangle as seen in Appendix J, this Sierpinski triangle is the result of 500,000 iterations of a simple rule that relies on randomness. This represents one of the seemingly contradictory relationships in chaos theory between randomness and structure.