Modeling A Soap Bubble Rupture

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Phantom v9.1 High Speed Camera

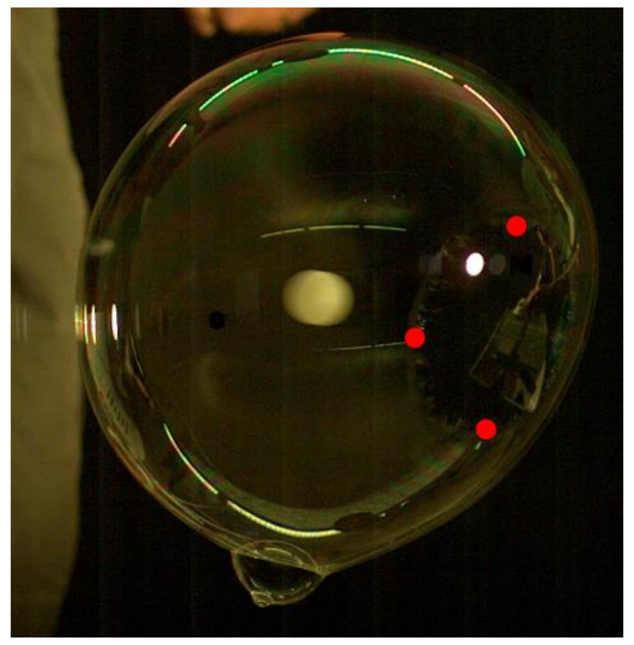
14-bit Color Camera
3 GB of RAM

Full Resolution:
1632 x 1200 Pixels
1000 FPS
0.88 Seconds

Introduction

A soap bubble was dropped directly in front of projectile launcher. A solid sphere was then launched through the bubble while the high-speed camera filmed the event. The projectile entered then exited the bubble before it had fully popped. Our goal was to determine the rate of expansion for both the entrance and exit holes and produce a 3D representation of the event.

Data Collection

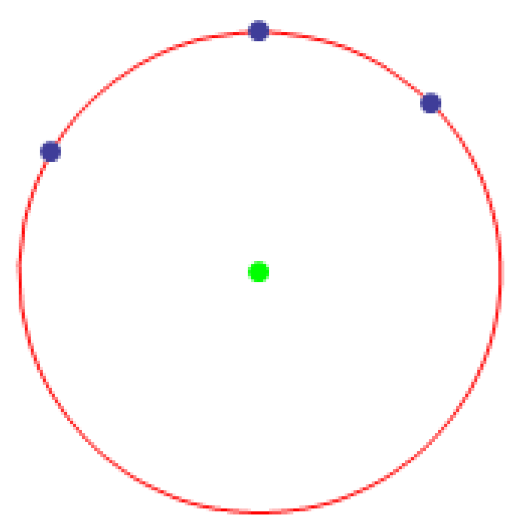


For each frame, three data points were collected along the outer edge of the puncture hole(s) along with the center location of the bubble.

After acquiring the data, it was decided that first solving a simpler problem would assist in solving the more complex problem of modeling the bubble rupture in three dimensions.

Fitting a Circle in Two Dimensions

Given three non-colinear points it is possible to determine the equation of a circle intersecting the points:



$$(x_1 - x_c)^2 + (y_1 - y_c)^2 = r^2$$
$$(x_2 - x_c)^2 + (y_2 - y_c)^2 = r^2$$
$$(x_3 - x_c)^2 + (y_3 - y_c)^2 = r^2$$

The radius can be eliminated:

$$(x_1 - x_c)^2 + (y_1 - y_c)^2 = (x_3 - x_c)^2 + (y_3 - y_c)^2$$
$$(x_2 - x_c)^2 + (y_2 - y_c)^2 = (x_3 - x_c)^2 + (y_3 - y_c)^2$$

which allows for the solving of the x-center and y-center (E. Sheetz solved this by hand):

$$x_c = \frac{y_2x_1^2 - y_2x_3^2 + y_2y_1^2 - y_2y_3^2 - y_3x_1^2 + y_3x_3^2 - y_3y_1^2 + y_3^2 - y_1x_2^2 + y_1x_3^2 - y_1y_2^2 + y_1y_3^2 + y_3x_2^2 - y_3x_3^2 + y_3y_2^2 - y_3^3}{2x_1y_2 - 2x_1y_3 - 2x_2y_1 + 2x_2y_3 + 2x_3y_1 - 2x_3y_2}$$

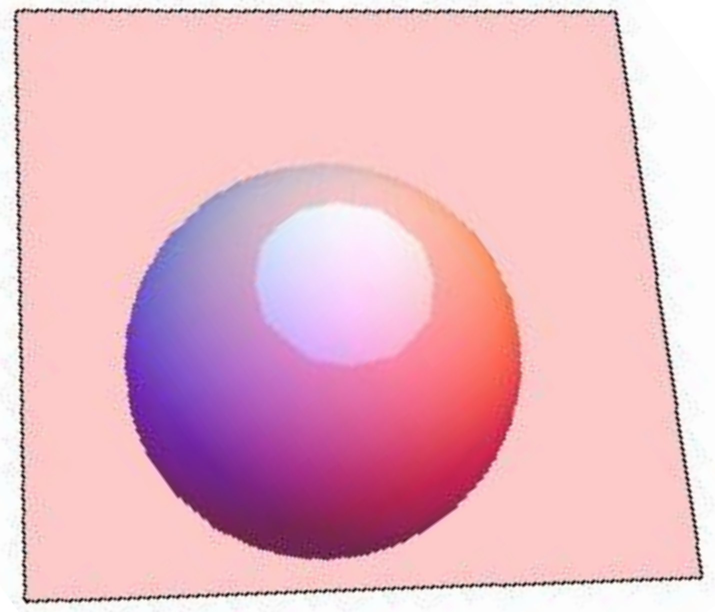
$$y_c = \frac{x_2x_1^2 - x_2x_3^2 + x_2y_1^2 - x_2y_3^2 - x_3x_1^2 + x_3^3 - x_3y_1^2 + x_3y_3^2 - x_1x_2^2 + x_1x_3^2 - x_1y_2^2 + x_1y_3^2 + x_3x_2^2 - x_3^3 + x_3y_2^2 - x_3y_3^2}{-2x_1y_2 + 2x_1y_3 + 2x_2y_1 - 2x_2y_3 - 2x_3y_1 + 2x_3y_2}$$

Solution in Three Dimensions

After solving the 2D problem, the more difficult 3D problem could be attempted. Given three non-colinear points it is possible to determine the equation of a circle in 3D space intersecting the points:

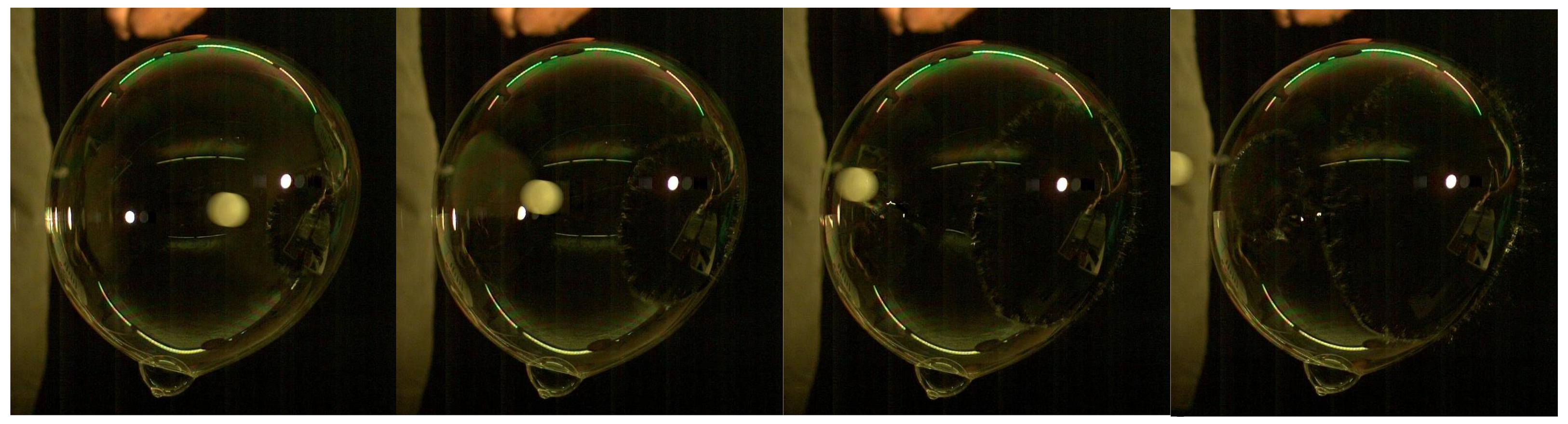
$$(x_1 - x_c)^2 + (y_1 - y_c)^2 + (z_1 - z_c)^2 = r^2$$
$$(x_2 - x_c)^2 + (y_2 - y_c)^2 + (z_2 - z_c)^2 = r^2$$
$$(x_3 - x_c)^2 + (y_3 - y_c)^2 + (z_3 - z_c)^2 = r^2$$

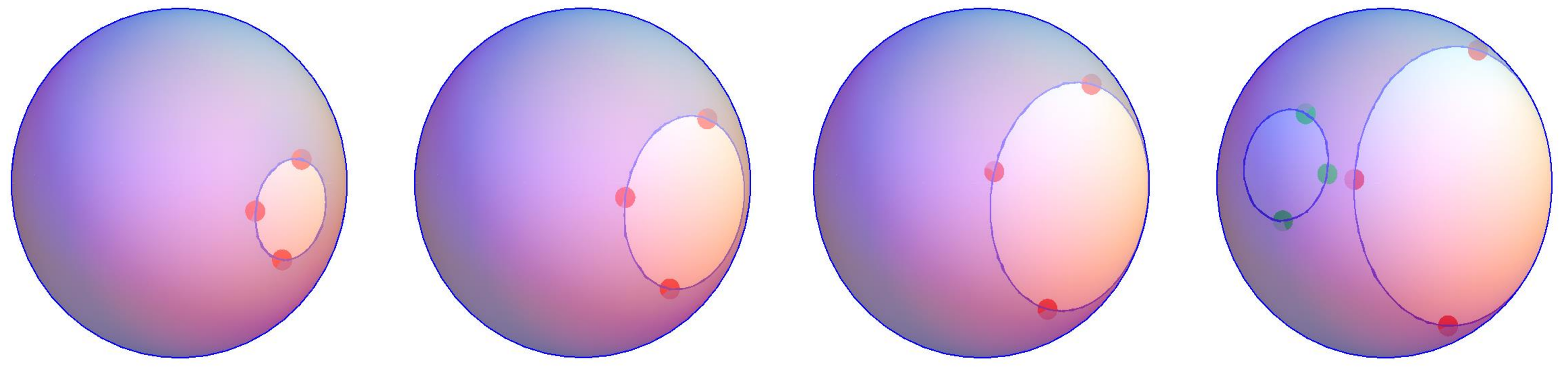
With the further stipulation that all three points must lie in the same plane, *Mathematica* 9 was used to solve the resulting system of equations for the circle's center and radius.



Graphically, the circle is the intersection between a sphere and a plane.

The data points collected for each frame were used to produce a 3D representation using this technique. The time between each image below is 6 ms.





The graph below shows the rate of expansion of the entrance hole of the bubble over time.