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The Development of Model Predictive Control in Automotive Industry: A Survey

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***Abstract*— Model Predictive Control (MPC) is an established control technique in chemical process control, due to its capability of optimally controlling multivariable systems with constraints on plant and actuators. In recent years, the advances in MPC algorithms and design processes, the increased computational power of electronic control units, and the need for improved performance, safety and reduced emissions, have drawn considerable interest in MPC from the automotive industry. In this paper we survey the investigations of MPC in the automotive industry with particular focus on the developments at Ford Motor Company. First, we describe the basic MPC techniques used in the automotive industry, and the early exploratory investigations. Then we present three applications that have been recently prototyped in fully functional production-like vehicles, highlighting the features that make MPC a good candidate strategy for each case. We finally present our perspectives on the next challenges and future applications of MPC in the automotive industry.**

I. Introduction

The operation of automotive systems must be optimal throughout the entire operating range to address tightening fuel economy, emissions and safety requirements. As a consequence, applications in the automotive industry are constantly providing new opportunities for the use of control techniques that are capable of optimizing the system response while non-conservatively enforcing specification constraints.

Model Predictive Control [1] has been developed specifically for these purposes, in the domain of chemical and process control [2]. Typical process control problems are characterized by relatively long sampling periods (often measured in minutes and hours) and ample processing and memory resources. On the other hand, automotive control problems are facing severe and diametrically opposite constraints, where typical sampling periods are few milliseconds and on-board computing power is limited for packaging, cost and mobility reasons. However, in recent years the increase in processor speed and memory, and the development of new algorithms has enabled the application of MPC for automotive control problems. For such applications, MPC has several attractive features. First, MPC allows the design of multivariable feedback controllers with similar procedural complexity as of single variable ones. In addition, MPC allows for the specification in the design phase of constraints on system inputs, states, and outputs, which are then enforced by the controller. Furthermore, MPC allows for the specification of an objective function which is optimized by the controller. Other advantageous MPC features are the capability of dealing with time delays [3], of rejecting measured and unmeasured disturbances [4], and of taking advantage from future information [5]. Finally, there is philosophical attractiveness to MPC since it embodies both (receding horizon) optimization and feedback adjustment, thus mimicking many processes in nature that seem to inherently operate in this way.

MPC has been investigated in several automotive control applications, including engine [3], [6]–[9], transmission [10], [11], emissions [12], [13], mechatronics actuators [14], steering [5], [15], suspensions [16]–[18], energy management [19], [20], thermal management [21]. Several companies have contributed and supported MPC research, including Ford, BMW, Honda, Honeywell, PSA, Toyota. At Ford Motor Company, control technologies based on MPC have been explored since the early 1990s inspired by the landmark survey by Professor Morari and his coworkers [1]. Some of the results from these early efforts were later published in [6] in the context of Idle Speed Control, where the advantages of MPC – such as the ability to handle actuator constraints and load preview – were demonstrated. In the following years several other exploratory investigations were developed including traction control [22], semi-active suspensions [18], and Direct Injection Stratified Charge (DISC) engine [7]. While these investigations were mostly limited to simulation studies, significant insight was obtained in benchmarking MPC capabilities within the automotive environment, and in some cases this even led to analytical breakthroughs, such as providing an explicit solution for optimal semi-active suspension regulation [18]. During the last decade, a number of MPC applications were developed in fully drivable prototype vehicles at Ford, with controller specifications in some cases very similar to the ones in production vehicles. Examples include traction control [22], autonomous vehicle and stability control [4], [5], [23], idle speed control [3], [24], and series HEV energy management [25]. While these MPC controllers still required significant computational resources, the combined use of multiparametric programming [26], and appropriate design steps [3], [27] resulted in computational load within the reach of automotive microcontrollers.

In this paper we survey the progress in developing MPC controllers in automotive applications focusing primarily on the work done at Ford Motor Company. We first review the core of the MPC technology that was used in the

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MPC applications development (Section II), then we discuss the early exploratory investigations (Section III), mostly simulation-based, developed at Ford. In Section IV we survey three recent Ford applications of MPC, and we discuss the key features that make MPC appealing for each. Finally, in Section V we briefly discuss our perspective on the future challenges for MPC applications to automotive systems.

II. Model Predictive Control

Model predictive control [1] has been developed to integrate the performance of optimal control with the robustness of feedback control. Similar to optimal control, MPC selects the actions by optimizing a cost function while accounting for system dynamics and constraints. The general MPC optimal control problem is

$$\min_{U(t)} \quad F(x(N|t)) + \sum_{k=0}^{N-1} L(x(k|t), y(k|t), u(k|t)) \quad (1a)$$

$$\text{s.t.} \quad x(k+1|t) = f(x(k|t), u(k|t)), \quad (1b)$$

$$y(k|t) = h(x(k|t), u(k|t)), \quad (1c)$$

$$x_{\min} \leq x(k|t) \leq x_{\max}, \quad k = 1, \ldots, N_c, \quad (1d)$$

$$y_{\min} \leq y(k|t) \leq y_{\max}, \quad k = 0, \ldots, N_c, \quad (1e)$$

$$u_{\min} \leq u(k|t) \leq u_{\max}, \quad k = 0, \ldots, N_{cu}, \quad (1f)$$

$$x(0|t) = x(t), \quad (1g)$$

$$u(k|t) = \kappa(x(k|t)), \quad k = N_u, \ldots, N-1, \quad (1h)$$

where t is the discrete time index and for a vector v, the notation $v(h|t)$ denotes the value of v predicted h steps ahead from t, based on information up to t. Equations (1b), (1c) are the discrete time model of the system dynamics with sampling period T_s, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ are the system state, input, and output, respectively. The model is initialized at the current state estimate $x(t)$ by (1g). The optimizer in (1) is the control input sequence $U(t) = (u(0|t), \ldots, u(N-1|t))$, where N is the horizon. The cost function (1a) represents the performance objective, and it is composed of a stage cost L, and a terminal cost F. The constraints on states and outputs, and inputs are enforced along the horizons N_c and N_{cu}, respectively. The control horizon N_u is the number of optimized steps, before terminal control law (1h) is applied.

Based on optimal control problem (1), at any control cycle t the MPC control strategy operates as follows: (i), the current state estimate $x(t)$ is acquired and used to initialize (1) by (1g). (ii), problem (1) is solved thus obtaining an optimal input sequence $U^*(t)$. (iii), the first element of the optimal input sequence is applied to the system input, $u(t) = u^*(0|t)$. At the following cycle the process is repeated using the newly acquired state estimate, thus implementing feedback.

A. Common assumptions on dynamics and cost function

While the MPC strategy based on (1) is fairly general, some restrictions have to be imposed in order to obtain an optimization problem that can be solved in practice.

The cost function (1a) is formulated in terms of the weighted norm of the states, inputs, and outputs

$$\|x(N|t)\|_P^p + \sum_{k=0}^{N-1} \|x(k|t)\|_Q^p + \|u(k|t)\|_R^p + \|y(N|t)\|_S^p, \quad (2)$$

where Q, R, S, P are weight matrices, $p = 1, 2, \infty$. For $p = 1, \infty$, $\|v\|_Q^p$ is the p-norm of Qv, and, with a little abuse of notation, for $p = 2$, $\|v\|_Q^p = v'Qv$, $Q \geq 0$. For $p = 1, \infty$, (2) is formulated by linear equations and linear constraints, while for $p = 2$ by a convex quadratic equation.

Linear Time Invariant (LTI) dynamics are normally used for the prediction model, so that (1b), (1c) results in

$$x(k+1|t) = Ax(k|t) + Bu(k|t), \quad (3a)$$

$$y(k|t) = Cx(k|t) + Du(k|t). \quad (3b)$$

The outputs in (1) and (3) can formulate cost function terms and constraints on multiple states and/or inputs.

Using (2) and (3), problem (1) results in [26], [28]

$$\min_{(U(t),\varepsilon(t))} \quad U'(t)\mathcal{Q}U(t) + x(t)\mathcal{C}'U(t) + \mathcal{F}\varepsilon(t), \quad (4a)$$

$$\text{s.t.} \quad \mathcal{G}U(t) \leq \mathcal{W} + \mathcal{S}x(t), \quad (4b)$$

$$\mathcal{G}_e U(t) + \mathcal{T}\varepsilon(t) \leq \mathcal{W}_e + \mathcal{S}_e x(t), \quad (4c)$$

where $\varepsilon(t) \in \mathbb{R}^{n_e}$ is a vector of auxiliary variables used when $p = 1, \infty$, while $n_e = 0$ for $p = 2$. Setting $p = 1, \infty$ in (2), results in $\mathcal{Q} = 0$, $\mathcal{C} = 0$, hence in a Linear Program (LP). For $p = 2$, (4) is a Quadratic Program (QP). Algorithms with guaranteed convergence for solving LP and QP exist.

More recently [26], it was shown that the solution of (4) as a function of $x(t)$ can be computed by parametric programming, thus synthesizing the MPC state feedback law

$$u(t) = F_{i(t)}x(t) + G_{i(t)}, \quad (5a)$$

$$i(t) : H_{i(t)}x(t) \leq K_{i(t)}, \quad (5b)$$

$$i(t) \in \{1, \ldots, s\}.$$

The state space is partitioned by (5b) into s polyhedral regions (indexed by i), and (5a) assigns to each region an affine control law. Indeed, real time optimization is not needed since, the MPC control law is evaluated by: 1) finding the region $i(t) = \bar{\imath}$ such that all the inequalities constraints (5b) are satisfied, and 2) evaluating the gain (5a) for $i(t) = \bar{\imath}$ and use the result as input. Using (5), the closed-loop dynamics is computed, and stability and other properties can be analyzed [24].

B. Hybrid, switched, and time varying models

Several extensions to the assumptions in Section II-A have enlarged the applicability of MPC. LTI dynamics (3) can be substituted by the piecewise affine dynamics

$$x(k+1|t) = A_{i(k|t)}x(k|t) + B_{i(k|t)}u(k|t) + \phi_{i(k|t)}, \quad (6a)$$

$$y(k|t) = C_{i(k|t)}x(k|t) + D_{i(k|t)}u(k|t) + \zeta_{i(k|t)}, \quad (6b)$$

$$i(k|t) : H^x_{i(k|t)}, x(k|t) + H^u_{i(k|t)}u(k|t) \leq K^x_{i(k|t)}, \quad (6c)$$

$$i(t) \in \{1, \ldots, s\},$$

where (6c) partitions the state-input space into polyhedral regions, and (6a), (6b) assign to each region a state update and output equation, respectively. PWA systems [29] represent a powerful formalism which has been shown to be equivalent, under mild assumptions, to several other classes of systems [30]. By leveraging such equivalencies, the MPC problem (1) where (1b), (1c) are formulated as (6), and (1a) as (2) for $p = 1, \infty$ ($p = 2$) results in a Mixed Integer Linear (Quadratic) Program, MILP (MIQP) [31]. The explicit feedback law calculation is still possible for MILP, while only a semi-explicit calculation is possible for MIQP [32]. However, in both cases the complexity of the feedback law (5) tends to be large [15].

The switched MPC strategy mitigates this problem. In switched MPC, the piecewise region index is updated only at the initial step of the prediction horizon. For partitions that do not depend on the current input, s PWA control laws (one per region) are computed and merged, as opposed to s^N for (6), resulting in a much simpler control law (5).

A further extension is to use as prediction model the linear time varying (LTV) dynamics

$$x(k+1|t) = A(t)x(k|t) + B(t)u(k|t) + \phi(t), \quad (7a)$$
$$y(k|t) = C(t)x(k|t) + D(t)u(k|t) + \zeta(t), \quad (7b)$$

where the system matrices can change at any control cycle. Model (7) is obtained for instance when the nonlinear dynamics (1b), (1c) are linearized [5], [33] around the current state estimate, $x(t)$. While indeed powerful, the LTV system (7) changes the parameters of the optimization problem at every cycle. Hence, (4) is continuously reconstructed, and the calculation of feedback law in the form (5) is, in general, not possible.

III. Early Exploratory Investigations

In this section we briefly summarize some of the early MPC applications in the automotive area, primarily developed by Ford researchers and their academic partners, aimed at assessing the potential benefits of MPC for automotive applications.

A. Powertrain Control

Idle Speed Control (ISC) is one of the most basic and representative automotive control problems [34], and still one of the most important aspects of engine operation since typical vehicles spend a significant amount of time and fuel at idle [6]. The main objective of ISC is to keep the engine speed under tight and robust control with target speed as low as possible, for fuel economy, while preventing engine stalls. The capability of MPC for dealing with constrained time delay systems are well highlighted in ISC. In [6], an MPC based ISC was designed with air bypass valve u_{thr} and spark timing, u_{spk}, as control inputs, both constrained and affected by time delay. The time delay on the airflow δ_a is long and caused by physical phenomena, while the delay on the spark channel δ_s is smaller and mostly computational. However, the spark has smaller steady-state authority than the airflow over the engine torque.

The engine speed dynamics [35] are described by

$$G_{\text{thr}}(s) = k_1 \frac{1}{\frac{s^2}{\omega_1^2} + 2\frac{\zeta_1}{\omega_1}s + 1}, \quad (8a)$$
$$G_{\text{spk}}(s) = k_2 \frac{\frac{s}{a} + 1}{\frac{s^2}{\omega_2^2} + 2\frac{\zeta_2}{\omega_2}s + 1}, \quad (8b)$$
$$Y(S) = G_{\text{thr}}(s)e^{-\delta_a s}U_{\text{thr}}(s) + G_{\text{spk}}(s)e^{-\delta_s s}U_{\text{spk}}(s), \quad (8c)$$

where $Y(s)$ is the Laplace transform of the engine speed $\xi(t)$. The transfer functions (8a), (8b) sampled with sampling period $T_s = 20$ms were used as MPC prediction model.

The ISC inputs are constrained, and the difference between the engine speed and reference r_ξ has to remain bounded, in order to avoid the possibility of engine stalls and flares,

$$e^{\min} \leq \xi(k) - r_\xi(k) \leq e^{\max}, \quad (9a)$$
$$u_{\text{thr}}^{\min} \leq u_{\text{thr}}(k) + u_{FF}(k) \leq u_{\text{thr}}^{\max}, \quad (9b)$$
$$u_{\text{spk}}^{\min} \leq u_{\text{spk}}(k) \leq u_{\text{spk}}^{\max}(k), \quad (9c)$$

where u_{FF} is the airflow feedforward which is approximately constant, and where $u_{\text{spk}}^{\max}$ is the upper bound on spark timing (due to knocking limit), which changes significantly with the engine conditions (e.g., temperature, load). In addition in [6], constraints on the control input variations (i.e., slew rate) were also considered. In order to guarantee regulation to the reference in presence of constant disturbances, and to recover full spark reserve in steady state, the MPC cost function is formulated as

$$\sum_{k=1}^{N} q_y(\xi(k|t) - r_\xi(t))^2 + q_i\varepsilon_i(k|t)^2 + q_{\text{spk}}u_{\text{spk}}(k|t)^2 + r_{\text{thr}}\Delta u_{\text{thr}}(k|t)^2 + r_{\text{spk}}\Delta u_{\text{spk}}(k|t)^2 \quad (10)$$

where Δu_{thr}, Δu_{spk} are the rate of change of the inputs, and $\varepsilon_i(k|t)$ is the integral action that ensures offset-free rejection of constant disturbances, i.e.,

$$\varepsilon_i(k+1|t) = \varepsilon_i(k|t) + T_s(\xi(k|t) - r_\xi(t)). \quad (11)$$

For the early investigations, a simplified form of (10) was used with the $q_i, q_{\text{spk}}, r_{\text{spk}} = 0$ and $u_{\text{spk}}^{\max}(k) = u_{\text{spk}}^{\max}$. A detailed simulation study was performed for various MPC horizons and constraint/performance weights [6]. The results showed that compared with other techniques [34], such as robust μ-synthesis, MPC was especially effective in dealing with actuator constraints. In addition, it was suitable for a tuning that more directly influenced the key attributes and objectives. It was also found that a preview of known torque disturbances or loads, e.g., air conditioning, can substantially improve the ISC performance and reduce or even eliminate the need for spark control. For the considered 2.9L engine, only 0.5s preview of step-load change resulted in up to tenfold reduction in engine speed sag.

Another early automotive application of MPC was Traction Control (TC) [22]. TC enhances driver's ability to control a vehicle on slippery roads, by a tight and robust control of tire slip to prevent the loss of tractive and lateral force production capability. The TC dynamics typically include significant non-linearities, especially due to tires, which have been modeled as piecewise linear functions in the force-slip diagram leading to a hybrid MPC setting [22]. The simulation results demonstrated significant advantages of the hybrid MPC, which were subsequently confirmed through in-vehicle implementation of both linear and hybrid MPC executed with $T_s = 20$ms on a Pentium II laptop. Tests on icy surfaces demonstrated feasibility of MPC methodology and confirmed the superior performance of hybrid MPC versus linear MPC for traction control. In particular, the hybrid MPC controller with $p = 1$ in (2) led to up to a 20% reduction in peak slip deviation when compared to linear MPC. It is believed that this was the first in-vehicle implementation of hybrid MPC.

In [7], the application of hybrid MPC to mode switching control in Direct Injection Stratified Charge (DISC) engines was studied. The MPC coordinated electronic throttle, spark timing and EGR valve to enable torque and air-to-fuel ratio tracking and stratified to homogeneous mode transitions. Mode dependent constraints on air-to-fuel ratio and spark timing range were also enforced. The work represents one of the first applications of hybrid MPC to control a complex engine with constraints and different operating modes. Related ideas have been later explored for an advanced hybrid electric powertrain for Jaguar-Land Rover applications [19].

In addition to the above, a number of other exploratory MPC applications were evaluated at Ford. These include, among others, drivability enhancement [36], magnetic valve actuator control [14], and air-to-fuel ratio control [13].

B. Chassis Control

MPC has been considered to include road disturbance information in suspension control [16], [17]. Further, taking advantage of the recent advances in explicit MPC (5), it was established that the optimal control for semi-active suspensions exists in the form of an affine state feedback [18]. Prior to this discovery, it was not clear if an explicit optimal solution can be obtained for semi-active suspensions, which are usually modelled as a linear dynamic system with a nonlinear passivity constraint or as a bi-linear system with input constraints. The so-called "clipped optimal" control that "clips" the optimal solution of an active suspension is popular, but has been verified to be a sub-optimal solution [37]. The MPC effort towards semi-active suspension applications at Ford was the first to find the explicit solution and provided a benchmark against other sub-optimal control laws.

For autonomous vehicle development, MPC was applied for real time path following in [5], [23], where it was implemented with $T_s = 50$ms in a prototype vehicle with steering actuation. Thanks to the prediction based on current steering input and plant model, the MPC autonomous vehicle proved to be more robust for varying road surface friction than a specifically tuned steering robot [5]. In [23], the MPC controller for the autonomous vehicle was extended to actuate steering and braking. The MPC approaches implemented in experiments include models of varying complexities (from two-track models with control on four braking torques, to single track models with control on yaw moment, [5], [23]). With each model, both nonlinear model predictive control (NLMPC), and linear time varying model predictive control (LTVMPC) were developed and tested on a double lane change maneuver. While the NLMPC (1) uses a fully nonlinear model incorporating Pacejka tire model, LTVMPC linearizes the model at each sampling time (7) and identifies the cornering stiffness in real time. These studies demonstrated that the proposed algorithm: (i) coordinates the use of steering and braking, in presence of constraints, (ii) stabilizes the vehicle despite operating in a wide range of tire/road characteristics, (iii) can reproduce complex countersteering behavior performed by skilled drivers.

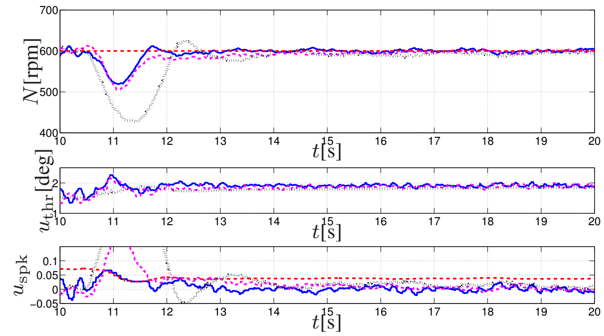


Fig. 1. Idle speed control by MPC. Disturbance rejection. Base controller (dot), single-input MPC (dash), multi-input MPC (solid).

IV. Recently Prototyped Applications

We now survey three applications of MPC in different areas of automotive control that were developed at Ford and became operational on fully drivable prototype vehicles.

A. Idle Speed Control: refinements, synthesis, vehicle tests

The ISC controller described in Section III-A has been recently extended, implemented in a prototype vehicle, and tested under real and extreme operating conditions [3], [24]. The extensions include the use of the full set of weights in (10) to optimally achieve the specifications, the use of the time-varying spark constraint in (9), and the use of electronic throttle and spark torque ratio as control commands, rather than bypass valve and spark timing. The MPC controller for ISC is synthesized in explicit form (5) with $T_s = 30$ms, and tested in a prototype vehicle with a 4.6L, V8 engine and a 4 speed automatic transmission.

Figure 1 shows a test where the power steering pump, which acts as a load on the crankshaft, is suddenly engaged at full power. The MPC that adjusts both throttle and spark is compared with a base controller, which has two separated error-based feedback loop (PIDs) with feedforward, and with an MPC that actuates only the throttle yet retains the base controller for the spark channel. For the controllers that do not enforce spark constraints, the spark request is saturated a posteriori. Since MPC accounts for time delays and provides nonlinear control action induced by the constraints and idle

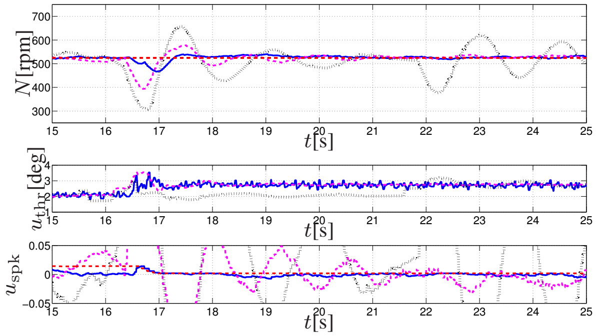


Fig. 2. Idle speed control by MPC. Double disturbance rejection. Base controller (dot), single-input MPC (dash), multi-input MPC (solid).

speed tracking error, higher control gains can be achieved, and disturbance rejection capabilities are improved.

Figure 2 reports a more challenging test where, after full power engagement of the power steering pump, the AC compressor is also engaged at full power. Since the power steering load brings the engine closer to knocking, when the second disturbance (AC compressor) hits, the controller has very limited spark authority. The MPC that controls throttle and spark can compensate the reduced spark authority by a more aggressive throttle action, hence maintaining good performance. Instead, the other controllers show a significant performance degradation since they are unaware of the loss of spark authority.

Due to the improved control performance the spark reserve can be reduced [24], hence improving the fuel economy at idle by approximately 4.5% in drive, and 6.5% in neutral.

B. Active front steering and differential braking: coordination of constrained actuators

The capability of MPC for coordinating multiple constrained actuators to achieve multiple goals has shown a positive impact in coordinating active front steering (AFS) and differential braking in vehicle stability control [4], [15]. In this application, the controller actuates the AFS and the brakes to simultaneously achieve yaw rate tracking and vehicle stabilization, which are sometimes conflicting goals.

The prediction model is the bicycle vehicle dynamics model with respect to the the front and rear tire slip angles α_f, α_r,

$$\begin{aligned}\dot{\alpha}_f &= \frac{F_f+F_r}{mv_x}-\frac{v_x}{a+b}(\alpha_f-\alpha_r+\delta_{\text{drv}}+\delta_{\text{AFS}}),\\ &\quad +\frac{a}{v_xI_z}(aF_f-bF_r+Y)-\varphi_{\text{drv}}-\varphi_{\text{AFS}}, &(12a)\\ \dot{\alpha}_r &= \frac{F_f+F_r}{mv_x}-\frac{v_x}{a+b}(\alpha_f-\alpha_r+\delta_{\text{drv}}+\delta_{\text{AFS}}),\\ &\quad -\frac{b}{v_xI_z}(aF_f-bF_r+Y), &(12b)\\ \dot{\delta}_{\text{AFS}} &= \varphi_{\text{AFS}}, &(12c)\\ \dot{\delta}_{\text{drv}} &= \varphi_{\text{drv}}, &(12d)\\ r &= \frac{v_x}{a+b}(\alpha_f-\alpha_r+\delta_{\text{drv}}+\delta_{\text{AFS}}), &(12e)\end{aligned}$$

where δ_{drv}, δ_{AFS} are the driver and AFS steering angles, respectively, φ_{drv}, φ_{AFS} are the corresponding steering rates, and $\delta=\delta_{\text{drv}}+\delta_{\text{AFS}}$ is the total road wheel angle. In (12), the longitudinal vehicle speed v_x, the vehicle mass m, the vehicle inertia along the vertical axis I_z, and the distances of the front and rear axes from the center of mass, a and b, are known and constant. Y is the yaw moment along the vertical axis that is obtained by applying different braking torques at different wheels, and r is the vehicle yaw rate. The forces F_f and F_r are the front and rear tire forces, which can be modelled as piecewise affine functions,

$$F_j(\alpha_j)=\begin{cases} d_j(\alpha_j+p_j)-e_j & \text{if} \quad \alpha_j<-p_j,\\ c_j\alpha_j & \text{if} \quad -p_j\leq\alpha_j\leq p_j,\\ d_j(\alpha_j-p_j)+e_j & \text{if} \quad \alpha_j>p_j\ ,\end{cases} \tag{13}$$

for $j\in\{f,r\}$ where p_j are the saturation angles, that model the three regions of operation of each tire pair (i.e., linear, positive, and negative saturation). The full steering vehicle dynamics is modelled by (12) and (13) thus resulting in a PWA system (6), which is converted to discrete time and formulated in switched form [15] to reduce complexity.

Constraints are enforced on slip angles and actuators,

$$\begin{aligned}\alpha_{f,\min}\leq\alpha_f\leq\alpha_{f,\max},\quad \alpha_{r,\min}\leq\alpha_r\leq\alpha_{r,\max}, &\quad (14a)\\ \delta_{\min}\leq\delta_{\text{AFS}}\leq\delta_{\max},\quad Y_{\min}\leq Y\leq Y_{\max}, &\quad (14b)\\ \varphi_{\min}\leq\varphi_{\text{AFS}}\leq\varphi_{\max}. &\quad (14c)\end{aligned}$$

The cost function encodes the multiple problem objectives, to track the driver-desired yaw rate $\hat{r}$, to maintain the vehicle "stable" (i.e., to keep the slip angles small), and to minimize the use of the brakes and of the steering rate,

$$\begin{aligned}&\sum_{k=0}^{N-1} q_i^{(r)}(r(k|t)-\hat{r}(t))^2+q_i^{(\varphi)}\varphi_{\text{AFS}}(k|t)^2\\ &+q_i^{(Y)}Y(k|t)^2+q_i^{(\alpha_f)}\alpha_f(k|t)^2+q_i^{(\alpha_r)}\alpha_r(k|t)^2. \quad (15)\end{aligned}$$

The feedback law of the MPC controller formulated on (12), (13) and on (14), (15), sampled with $T_s=50$ms, is tested in a prototype RWD vehicle, equipped with AFS, differential braking, and a precise localization system.

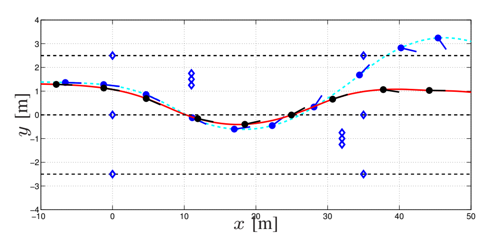
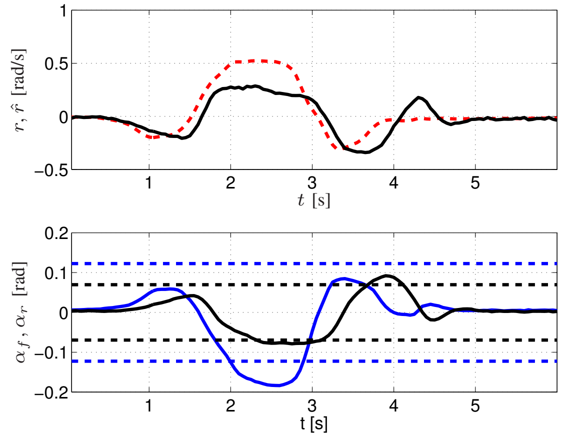
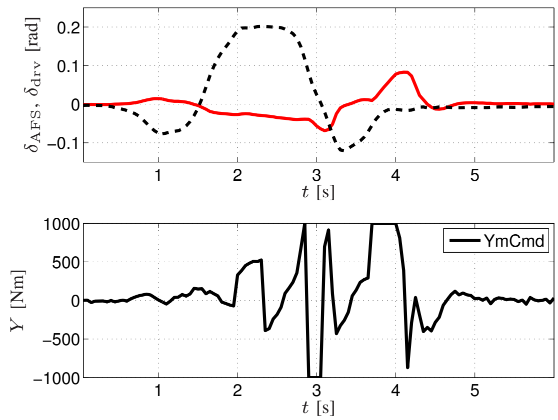


Fig. 3. Trajectories in a double lane change maneuver with (solid) and without (dash) MPC stability control. Vehicle center of mass (circle) and heading (line).

Among several tests executed, the capabilities of MPC are clear in the double lane change, see Figures 3, 4. In Figure 3 the trajectories obtained with and without the MPC controller are shown, where we observe that the double lane change



(a) Upper plot: Yaw rate reference (dash) and yaw rate (solid). Lower plot: Slip angles (solid) and saturation angles (dash). Front tire data in blue, rear in black.



(b) Upper plot: AFS actuator steering angle (solid), driver steering angle (dash). Lower plot: Yaw moment from differential braking.

Fig. 4. MPC stability control performance in a double lane change.

is not properly completed without the controller. For the test with MPC, the states, inputs, and outputs time histories are reported in Figure 4. Initially, the MPC applies only a small countersteering by AFS to keep the yaw rate as close as possible to the desired yaw rate, and the brakes are used to stabilize the vehicle. Starting from around $t =$ 3.5s, the AFS is actuated more aggressively to complete the maneuver, since the stability and the tracking objectives become coincident.

C. Series HEV power management: transient behavior optimization

An application that is closer to the classical use of MPC in chemical process control [2], namely, transient response optimization, is the energy management in a Series Hybrid Electric Vehicle (SHEV). In the SHEV, the electric motor is the only source of traction while the engine power feeds a generator. This converts the mechanical power into electrical power which is coupled with the battery power in a DC bus. It was shown in [25] that for the SHEV an optimal steady state engine-generator operating curve, ζ^*_{sys}, as a function of the desired generator power P_{gen}, can be found

$$\zeta^*_{\mathrm{sys}}(P_{\mathrm{gen}}) = \arg \max_{\omega_{\mathrm{eng}},\tau_{\mathrm{eng}}} \eta_{\mathrm{sys}}(\omega_{\mathrm{eng}}, \tau_{\mathrm{eng}}) \tag{16a}$$

$$\text{s.t.} \quad \eta_{\mathrm{gen}}\left(\frac{\omega_{\mathrm{eng}}}{\kappa}, \kappa\tau_{\mathrm{eng}}\right)\omega_{\mathrm{eng}}\tau_{\mathrm{eng}} = P_{\mathrm{gen}}, \tag{16b}$$

where ω_{eng}, τ_{eng} are the engine speed and the engine torque, η_{gen} and η_{sys} are the efficiency of the generator, and combined efficiency of the engine-generator cascade, as a function of speed and torque, respectively, and κ is the gear ratio between engine and generator. For a desired traction power, an optimal steady state operating point in curve (16) can be identified. However, the transitions from one point to another involve powertrain and battery dynamics and need to be carefully controlled to optimize fuel consumption. In particular, it was shown that it is advantageous to use the battery as a buffer to "smooth" the engine transients. At the same time, the operating range of battery power and battery state-of-charge have to be carefully enforced. Thus, MPC was chosen as the candidate control strategy due to its capability of optimizing the transient while enforcing operating constraints. MPC has been previously applied to energy management of different HEV configurations in [19], [33], explicitly aiming at the fuel flow minimization. However, due to the complexity of the model, either nonlinear or piecewise affine (6), the resulting controllers were too complex for vehicle implementation.

The MPC prediction model for SHEV energy management is composed of the battery state of charge (SoC) dynamics, identified from data as the (possibly switching) integrator,

$$SoC(k+1) = SoC(k) - \gamma P_{\mathrm{bat}}(k), \tag{17}$$

where P_{bat} is the battery power. In addition, the generator power (P_{gen}) dynamics (which uniquely assigns the engine dynamics) satisfies

$$P_{\mathrm{gen}}(k+1) = P_{\mathrm{gen}}(k) + \Delta P_{\mathrm{gen}}(k) + P_{\mathrm{xt}}(k), \tag{18}$$

where ΔP_{gen} is the generator power variation, and P_{xt} is a slack power to be used only during sudden accelerations.

The powerflow from/to the battery is assigned by

$$\begin{aligned} P_{\mathrm{bat}}(k) \quad &= \quad P_{\mathrm{bus}}(k) - P_{\mathrm{gen}}(k) - \Delta P_{\mathrm{gen}}(k) \\ &\quad + P_{\mathrm{brk}}(k) - P_{\mathrm{xt}}(k). \end{aligned} \tag{19}$$

where P_{bus} is the desired tractive power reported at the DC-bus, and P_{brk} is the non-regenerative brake power.

The constraints of the MPC problem enforce the operating ranges of the powertrain components,

$$SoC^{\min} \le SoC \le SoC^{\max}, \quad P^{\min}_{\mathrm{bat}} \le P_{\mathrm{bat}} \le P^{\max}_{\mathrm{bat}}, \tag{20}$$

$$\Delta P^{\min}_{\mathrm{gen}} \le \Delta P_{\mathrm{gen}} \le \Delta P^{\max}_{\mathrm{gen}}, \quad P^{\min}_{\mathrm{gen}} \le P_{\mathrm{gen}} \le P^{\max}_{\mathrm{gen}}, \tag{21}$$

and the cost function has the following form,

$$\begin{aligned} &\sum_{k=0}^{N-1} r_{\Delta}\Delta P_{\mathrm{gen}}(k|t)^2 + \rho(P_{\mathrm{brk}}(k|t)^2 + P_{\mathrm{xt}}(k|t)^2) \\ &+ s_{\mathrm{bat}}P_{\mathrm{bat}}(k|t)^2 + q_{SoC}SoC(k|t)^2 + q_{\eta}\tilde{\eta}^{-1}_{\mathrm{sys}}(P_{\mathrm{gen}}(k|t)). \end{aligned} \tag{22}$$

The cost function weights in (22) model the tradeoff of

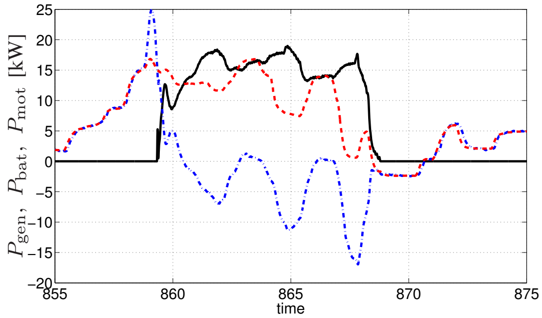


Fig. 5. Power smoothing effect of the MPC-based SHEV energy management for an interval of the City (UDDS) cycle experiment. Generator power (solid), battery power (dash dot), motor power (dash).

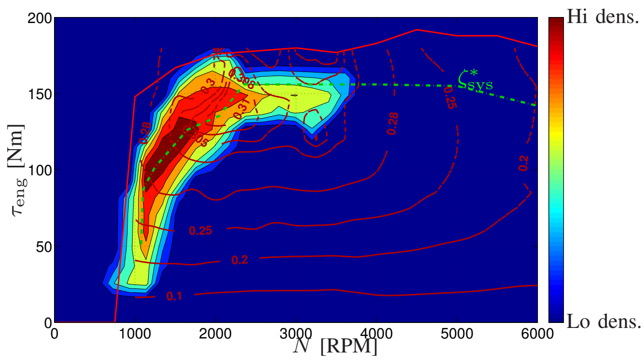


Fig. 6. MPC-based SHEV energy management in the City (UDDS) cycle experiment. Scatter plot of the engine operating points density.

objectives such as the reduction of the variation of the provided generator power (r_Δ), the stability of the battery state of charge around its setpoint (q_{SoC}), the reduction on battery powerflow ($s_{\rm bat}$), and also a slightly offsetting the steady state generator power to a point with higher efficiency, (q_η), based on an approximated quadratic inverse efficiency function ($\tilde{\eta}^{-1}$). The large weight ρ forces certain variables to 0, whenever possible.

Based on (17)–(22), a linear-quadratic MPC controller is obtained, for which the explicit feedback law (5) is computed. The controller has been implemented in the prototype vehicle *standard engine control unit,* and it has been evaluated for fuel consumption on a chassis-roll dynamometer. Figure 6 shows the distribution of the engine operating points in an experiment on the city (UDDS) driving profile, where the engine operating points are concentrated around the optimal curve $\zeta^*_{\rm sys}$. This is due to the MPC action that smoothen the transient. The behavior of the controller in reducing the engine transients can be noticed in Figure 5 where the generator power variations are significantly damped with respect to the tractive power request variations, with the battery used as a buffer. Repeated experimental tests have shown a fuel consumption reduction of about 5%, when compared to two baseline strategies.

V. Future Perspectives

In the coming years the number of multivariable automotive control applications is expected to increase as vehicles subsystems will be increasingly coordinated to improve fuel economy and safety. Thus, novel opportunities for MPC will emerge, including coordination of braking and powertrain in torque vectoring [38], coordination of engine and transmission [39] to improve fuel economy and responsiveness, control of complex engines such as GTDI and HCCI [40].

Being used at a supervisory level, more interaction of MPC with the driver is expected. Thus, a major research challenge for MPC will be to include a driver prediction model. This is already an ongoing effort. For instance, the AFS-braking controller described in Section IV-B has been recently extended with a more detailed prediction model of the driver steering behavior. This results in a controller that, while guaranteeing the same stability performance, is much more predictable and pleasant to drive. Similarly, a SHEV energy management strategy was recently proposed in [41], where the driver behavior is modelled as a Markov Chain learned in real-time, and used in a stochastic MPC algorithm. The resulting strategy adapts to the way the car is driven, to the drive cycle, and to the environment, achieving fuel economy close to the one obtained with future information.

MPC has shown large potential for use in automotive applications. However, there are also some fundamental challenges to overcome. First, calibrating MPC, similarly to other model-based approaches, can be complex. Some steps are being taken to reduce the calibration effort (see for instance [27], [42], and the references therein). Also, for several applications the MPC controllers are still too computationally complex, and low complexity explicit laws [43], [44] or fast approximated optimization algorithms [45] are necessary. Also, to guarantee the stability of MPC a-priori, without an excessive increase of the algorithm complexity is still challenging [39]. Finally, several applications have significant nonlinearities which complicate MPC design and implementation.

Overall, MPC is at the stage where it can be used in a number of automotive applications. The key to its widespread acceptance in the automotive industry is tightly linked to overcoming the few remaining challenges.

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References

[1] C. Garcia, D. Prett, and M. Morari, “Model predictive control: theory and practice–a survey,” *Automatica(Oxford)*, vol. 25, no. 3, pp. 335–348, 1989.

[2] S. Qin and T. Badgwell, “A survey of industrial model predictive control technology,” *Control Engineering Practice*, vol. 93, no. 316, pp. 733–764, 2003.

[3] S. Di Cairano, D. Yanakiev, A. Bemporad, I. Kolmanovsky, and D. Hrovat, "An MPC design flow for automotive control and applications to idle speed regulation," in *Proc. 47th IEEE Conf. on Dec. and Control*, 2008, pp. 5686–5691.

[4] S. Di Cairano and H. Tseng, "Driver-assist steering by active front steering and differential braking: Design, implementation and experimental evaluation of a switched model predictive control approach," in *Proc. 49th IEEE Conf. on Dec. and Control*, dec. 2010, pp. 2886 –2891.

[5] P. Falcone, F. Borrelli, J. Asgari, H. Tseng, and D. Hrovat, "Predictive active steering control for autonomous vehicle systems," *IEEE Trans. Contr. Systems Technology*, vol. 15, no. 3, pp. 566–580, 2007.

[6] D. Hrovat, "MPC-based idle speed control for IC engine," in *Proc. of FISITA conference*, Prague, Czech Rep., 1996.

[7] N. Giorgetti, G. Ripaccioli, A. Bemporad, I. Kolmanovsky, and D. Hrovat, "Hybrid Model Predictive Control of Direct Injection Stratified Charge Engines," *IEEE/ASME Transactions on Mechatronics*, vol. 11, no. 5, pp. 499–506, 2006.

[8] P. Ortner and L. del Re, "Predictive Control of a Diesel Engine Air Path," *IEEE Trans. Contr. Systems Technology*, vol. 15, no. 3, pp. 449–456, 2007.

[9] G. Stewart and F. Borrelli, "A Model Predictive Control Framework for Industrial Turbodiesel Engine Control," in *Proc. 47th IEEE Conf. on Dec. and Control*, Cancun, Mexico, Dec 2008, pp. 5704–5711.

[10] R. Amari, M. Alamir, and P. Tona, "Unified MPC strategy for idle-speed control, vehicle start-up and gearing applied to an Automated Manual Transmission," in *Proc. 17th IFAC World Congress*, Seoul, South Korea, 2008, pp. 7079–7085.

[11] T. Hatanaka, T. Yamada, M. Fujita, S. Morimoto, and M. Okamoto, "Explicit receding horizon control of automobiles with continuously variable transmissions," *Nonlinear Model Predictive Control*, vol. 384, pp. 561–569, 2009.

[12] R. Schallock, K. Muske, and J. Peyton Jones, "Model predictive functional control for an automotive three-way catalyst," *SAE Int. J. Fuels and Lubricants*, vol. 2, no. 1, pp. 242–249, 2009.

[13] S. Trimboli, S. Di Cairano, A. Bemporad, and I. Kolmanovsky, "Model predictive control for automotive time-delay processes: an application to air-to-fuel ratio," in *Proc. 8th IFAC Workshop on Time-delay Systems*, Sinaia, Romania, 2009, pp. 1–6.

[14] S. Di Cairano, A. Bemporad, I. Kolmanovsky, and D. Hrovat, "Model predictive control of magnetically actuated mass spring dampers for automotive applications," *Int. J. Control*, vol. 80, no. 11, pp. 1701–1716, 2007.

[15] S. Di Cairano, H. Tseng, D. Bernardini, and A. Bemporad, "Steering vehicle control by switched model predictive control," in *6th IFAC Symp. on Advances in Automotive Control*, Munich, Germany, 2010.

[16] R. Mehra, J. Amin, J. Hedrick, C. Osorio, and S. Gopalasamy, "Active suspension using preview information and model predictive control," in *Proc. IEEE Int. Conf. Control Applications*, Hartford,CT, 1997, pp. 860–865.

[17] M. Canale, M. Milanese, and C. Novara, "Semi-active suspension control using fast model-predictive techniques," *IEEE Trans. Contr. Systems Technology*, vol. 14, no. 6, pp. 1034–1046, 2006.

[18] N. Giorgetti, A. Bemporad, E. H. Tseng, and D. Hrovat, "Hybrid model predictive control application towards optimal semi-active suspension," *Int. J. Control*, vol. 79, no. 5, pp. 521–533, 2006.

[19] G. Ripaccioli, A. Bemporad, F. Assadian, C. Dextreit, S. Di Cairano, and I. Kolmanovsky, "Hybrid Modeling, Identification, and Predictive Control: An Application to Hybrid Electric Vehicle Energy Management," in *Hybrid Systems: Computation and Control*, ser. Lec. Not. in Computer Science. Springer, 2009, vol. 5469, pp. 321–335.

[20] H. Borhan, C. Zhang, A. Vahidi, A. Phillips, M. Kuang, and S. Di Cairano, "Predictive energy management of a power-split hybrid electric vehicle," in *Proc. 49th IEEE Conf. on Dec. and Control*, Atlanta, GA, 2010, pp. 4890–4895.

[21] C. Vermillion, J. Sun, and K. Butts, "Predictive control allocation for a thermal management system based on an inner loop reference model: Design, analysis, and experimental results," *IEEE Trans. Contr. Systems Technology*, vol. 19, no. 4, pp. 772 –781, 2011.

[22] F. Borrelli, A. Bemporad, M. Fodor, and D. Hrovat, "An MPC/hybrid system approach to traction control," *IEEE Trans. Contr. Systems Technology*, vol. 14, no. 3, pp. 541–552, 2006.

[23] P. Falcone, H. Tseng, F. Borrelli, J. Asgari, and D. Hrovat, "MPC-based yaw and lateral stabilisation via active front steering and braking," *Vehicle System Dynamics*, vol. 46, no. Supplement, pp. 611–628, 2008.

[24] S. Di Cairano, D. Yanakiev, A. Bemporad, I. V. Kolmanovsky, and D. Hrovat, "Model predictive idle speed control: Design, analysis, and experimental evaluation," *IEEE Trans. Contr. Systems Technology*, vol. 20, no. 1, pp. 84 –97, 2012.

[25] S. Di Cairano, W. Liang, I. Kolmanovsky, M. Kuang, and A. Phillips, "Engine power smoothing energy management strategy for a series hybrid electric vehicle," in *Proc. of the American Control Conference*, San Francisco, CA, 2011, pp. 2101–2106.

[26] A. Bemporad, M. Morari, V. Dua, and E. Pistikopoulos, "The Explicit Linear Quadratic Regulator for Constrained Systems," *Automatica*, vol. 38, no. 1, pp. 3–20, 2002.

[27] S. Di Cairano and A. Bemporad, "Model predictive control tuning by controller matching," *IEEE Trans. Automatic Control*, vol. 55, no. 1, pp. 185–190, jan. 2010.

[28] A. Bemporad, F. Borrelli, and M. Morari, "Model predictive control based on linear programming- the explicit solution," *IEEE Trans. Automatic Control*, vol. 47, no. 12, pp. 1974–1985, 2002.

[29] M. Johansson and A. Rantzer, "Computation of piece-wise quadratic Lyapunov functions for hybrid systems," *IEEE Trans. Automatic Control*, vol. 43, no. 4, pp. 555–559, 1998.

[30] S. Di Cairano and A. Bemporad, "Equivalent piecewise affine models of linear hybrid automata," *IEEE Trans. Automatic Control*, vol. 55, no. 2, pp. 498–502, 2010.

[31] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.

[32] A. Bemporad, F. Borrelli, and M. Morari, "On the optimal control law for linear discrete time hybrid systems," in *Hybrid Systems: Computation and Control*, ser. Lec. Not. in Computer Science. Springer-Verlag, 2002, vol. 2289, pp. 105–119.

[33] H. Borhan, A. Vahidi, A. M. Phillips, M. L. Kuang, I. V. Kolmanovsky, and S. Di Cairano, "MPC-based energy management of a power-split hybrid electric vehicle," *IEEE Trans. Contr. Systems Technology*, pp. 1 –11, 2011.

[34] D. Hrovat and J. Sun, "Models and control methodologies for IC engine idle speed control design," *Control Engineering Practice*, vol. 5, no. 8, pp. 1093 – 1100, 1997.

[35] S. J. Williams, D. Hrovat, C. Davey, D. Maclay, J. W. v. Crevel, and L. Chen, "Idle speed control design using an h-infinity appoach," in *Proc. American Contr. Conf.*, June 1989, pp. 1950–1956.

[36] D. Hrovat, M. Jankovic, I. Kolmanovsky, S. Magner, and D. Yanakiev, "Powertrain Control," in *The Control Handbook - Applications*. CRC Press, Taylor & Francis, 2011, vol. 5469, pp. 2.1 – 2.48.

[37] H. Tseng and J. Hedrick, "Semi-active control laws-optimal and sub-optimal," *Vehicle System Dynamics*, vol. 23, no. 1, pp. 545–569, 1994.

[38] D. Piyabongkarn, J. Lew, R. Rajamani, J. Grogg, and Q. Yuan, "On the use of torque-biasing systems for electronic stability control: Limitations and possibilities," *IEEE Trans. Contr. Systems Technology*, vol. 15, no. 3, pp. 581–589, 2007.

[39] C. Caruntu, A. Balau, M. Lazar, P. van den Bosch, and S. Di Cairano, "A predictive control solution for driveline oscillations damping," in *Hybrid Systems: Computation and Control*, Chicago, IL, 2011, pp. 181–190.

[40] J. Bengtsson, P. Strandh, R. Johansson, P. Tunestal, and B. Johansson, "Model predictive control of homogeneous charge compression ignition (hcci) engine dynamics," in *IEEE Int. Conf. on Control Applications*, Munich, Germany, 2006, pp. 1675 –1680.

[41] M. Bichi, G. Ripaccioli, S. Di Cairano, D. Bernardini, A. Bemporad, and I. Kolmanovsky, "Stochastic model predictive control with driver behavior learning for improved powertrain control," in *Proc. 49th IEEE Conf. on Dec. and Control*, Atlanta, GA, 2010, pp. 6077–6082.

[42] J. Garriga and M. Soroush, "Model predictive controller tuning via eigenvalue placement," in *Proc. American Contr. Conf.*, St. Louis, MO, 2008, pp. 429–434.

[43] C. Jones and M. Morari, "Approximate explicit MPC using bilevel optimization," in *Proc. European Control Conf.*, Budapest, Hungary, 2009, pp. 2396–2401.

[44] M. Canale, L. Fagiano, and M. Milanese, "Set membership approximation theory for fast implementation of model predictive control," *Automatica*, vol. 45, no. 1, pp. 45–54, 2009.

[45] Y. Wang and S. Boyd, "Fast model predictive control using online optimization," *IEEE Trans. Contr. Systems Technology*, vol. 18, no. 2, pp. 267–278, 2010.